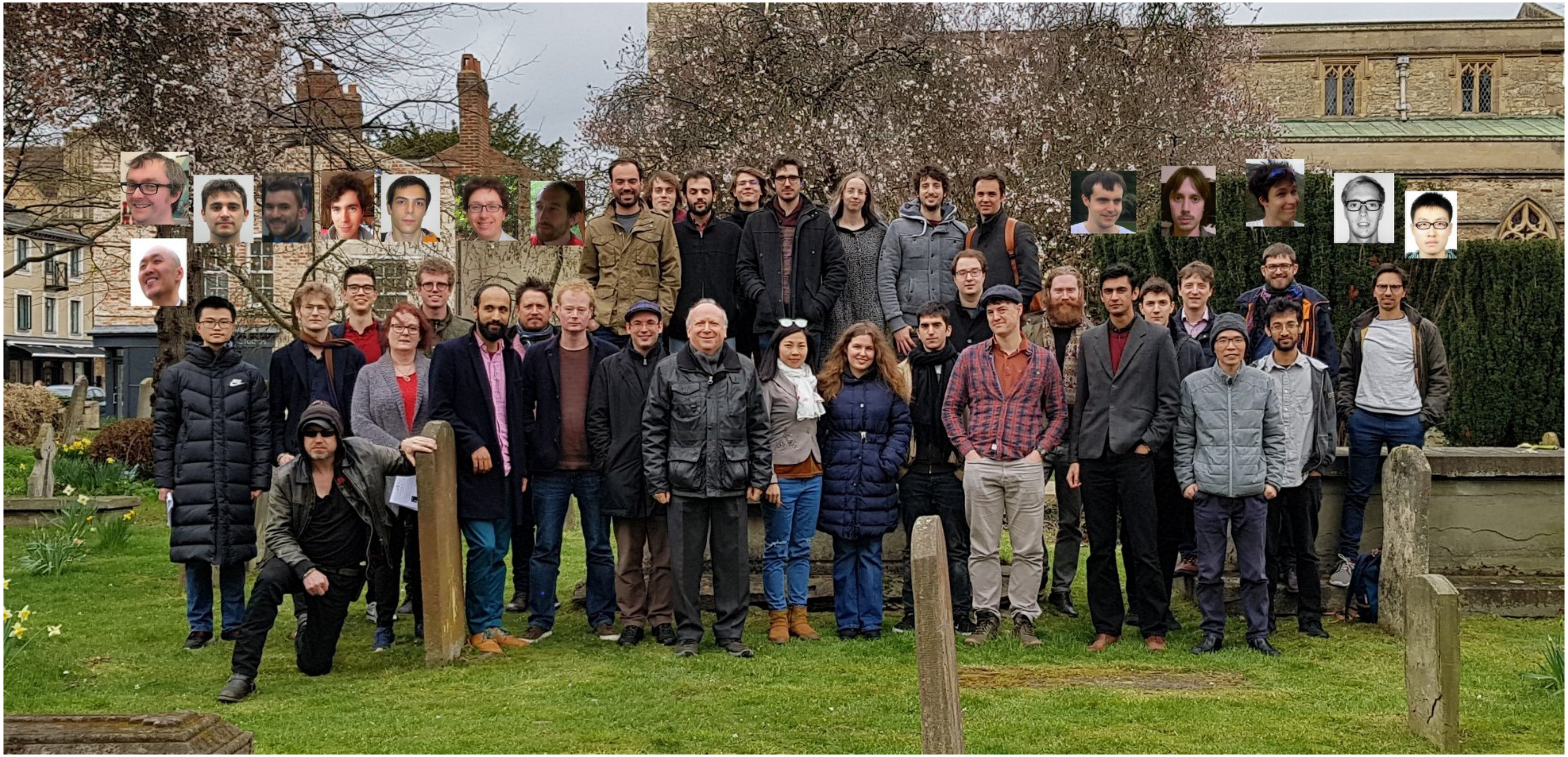
Bob Coecke

University of Oxford, CS

Cambridge Quantum Computing Ltd.

# CAUSALITY IN PICTURES



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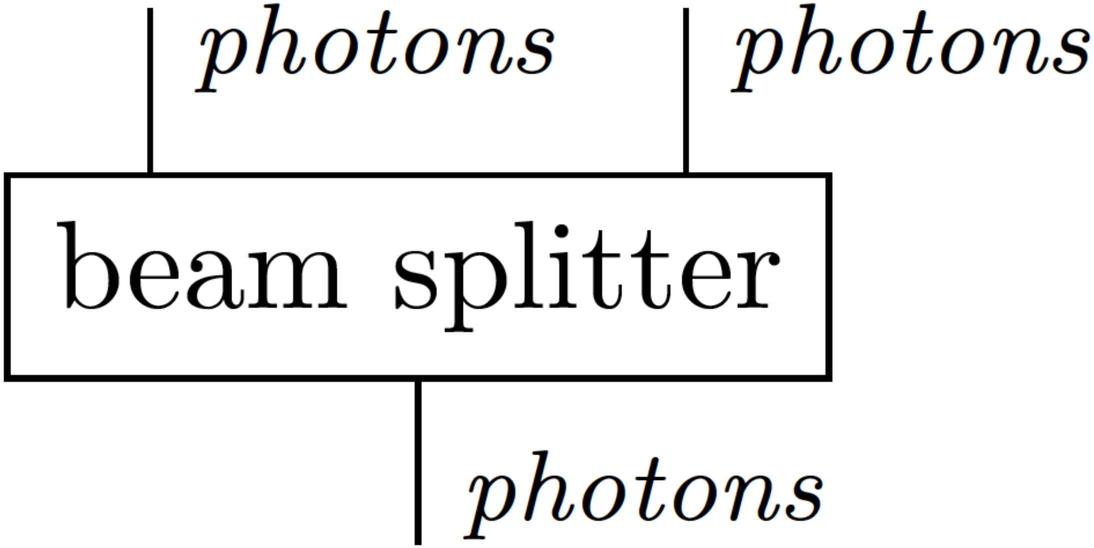
# wires and boxes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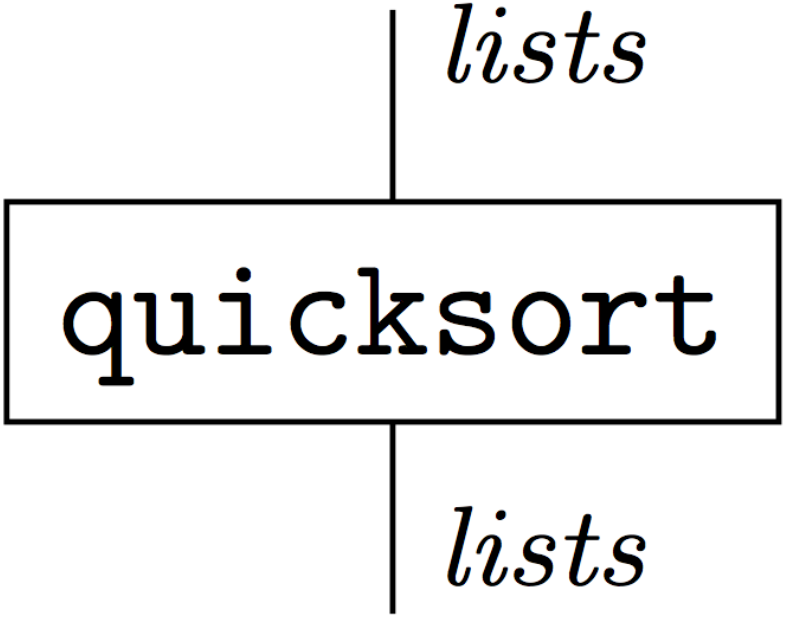
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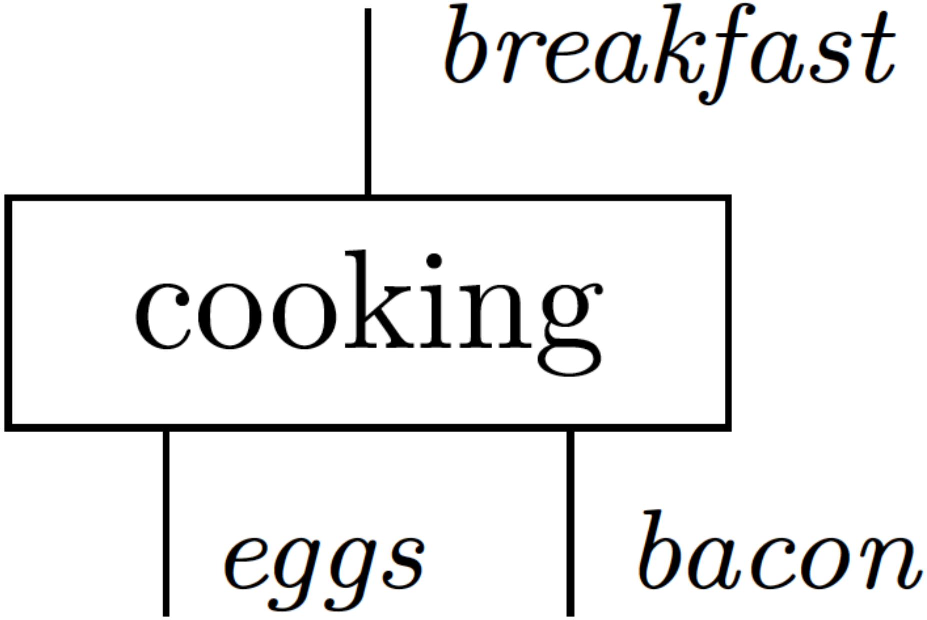
lists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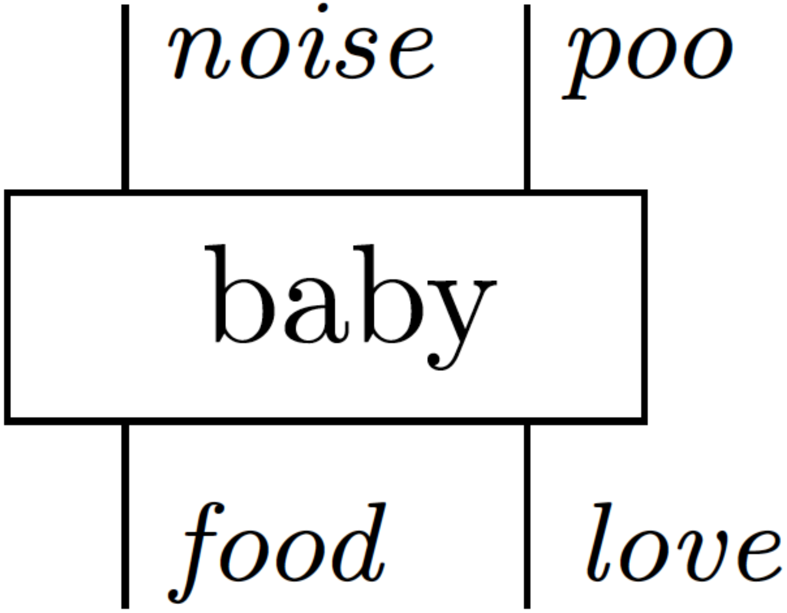
eggs
bacon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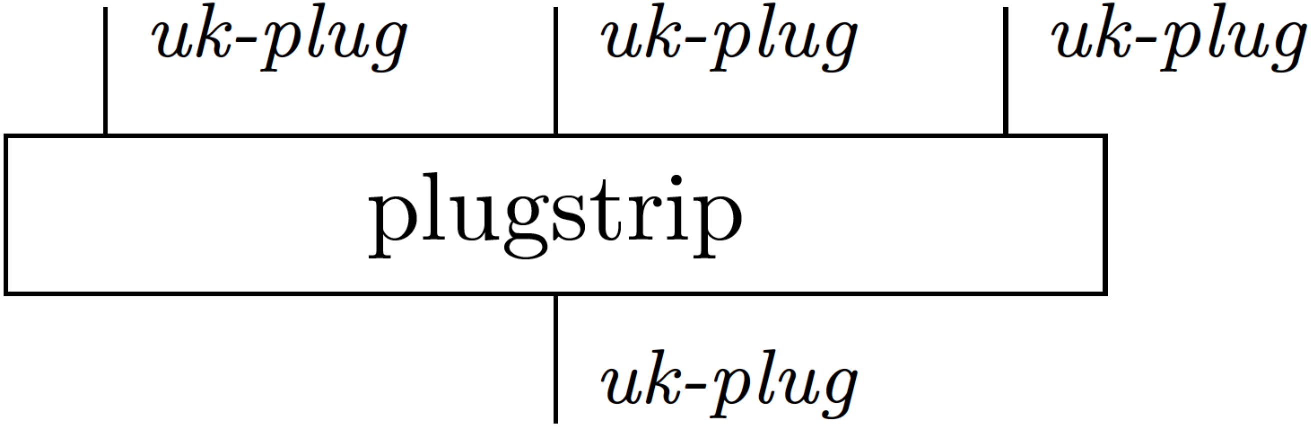
poo
baby
food
love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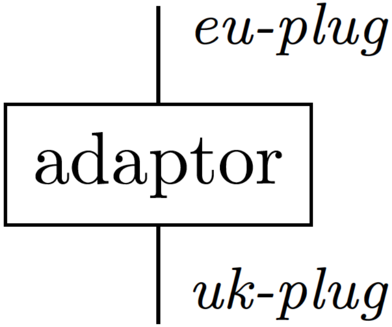
uk-plug
uk-plug
plugstrip
uk-plug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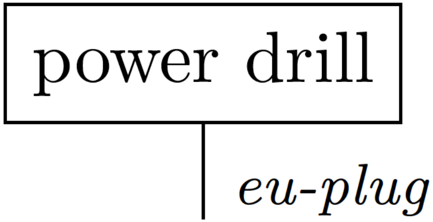
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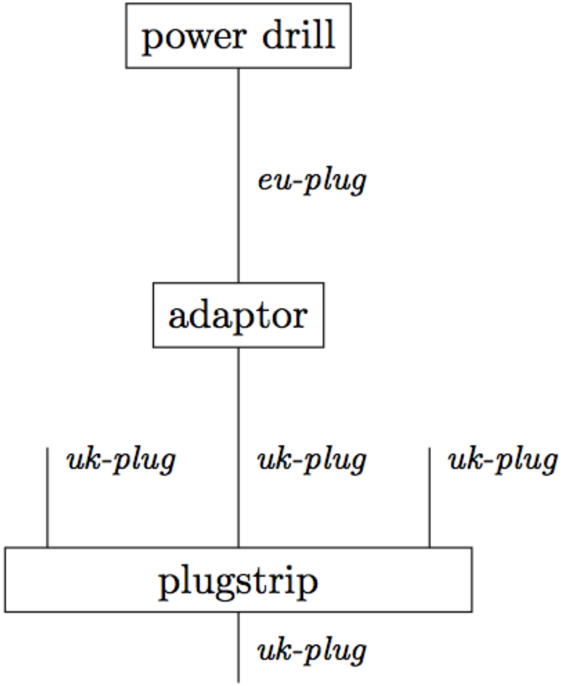
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eu-plug
adaptor
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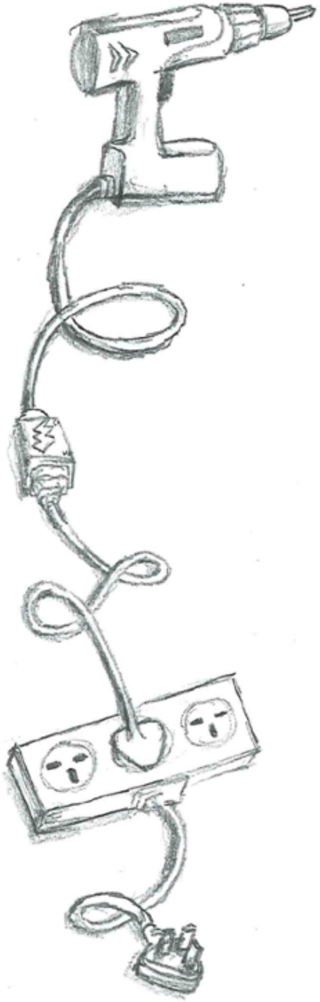
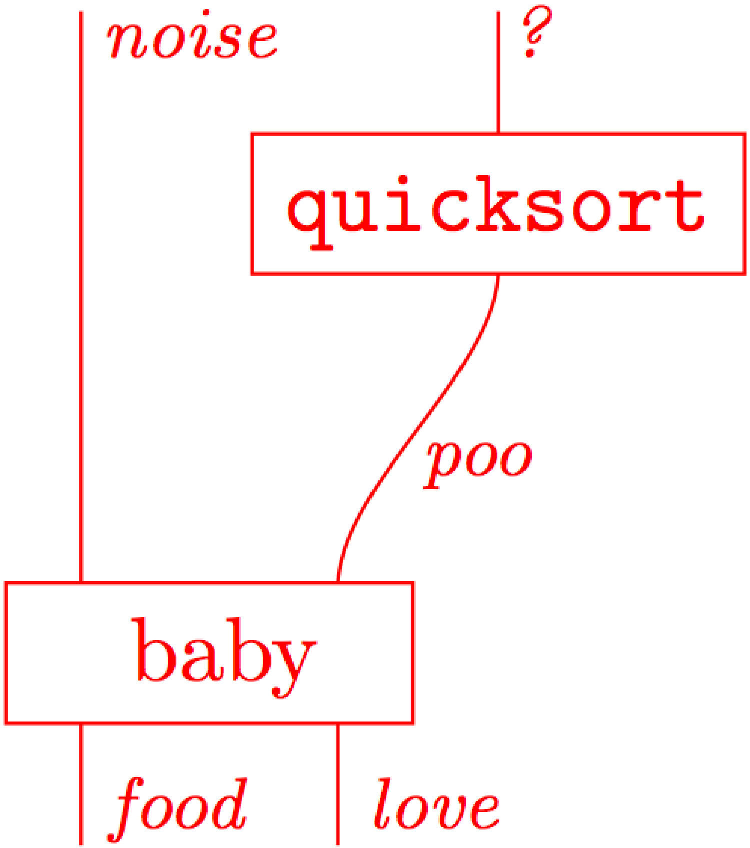
uk-plug
plugstrip
uk-plug

:=

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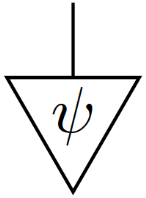
?
quicksort
poo
baby
food
love

State :=



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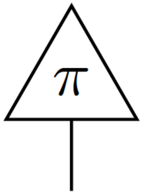

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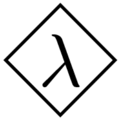

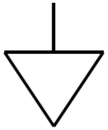
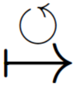
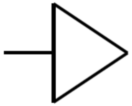
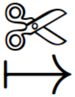

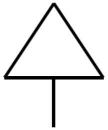
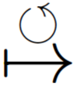
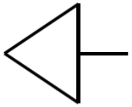
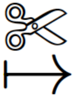

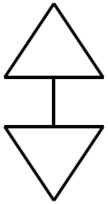
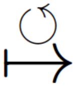
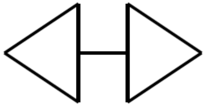
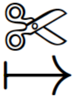

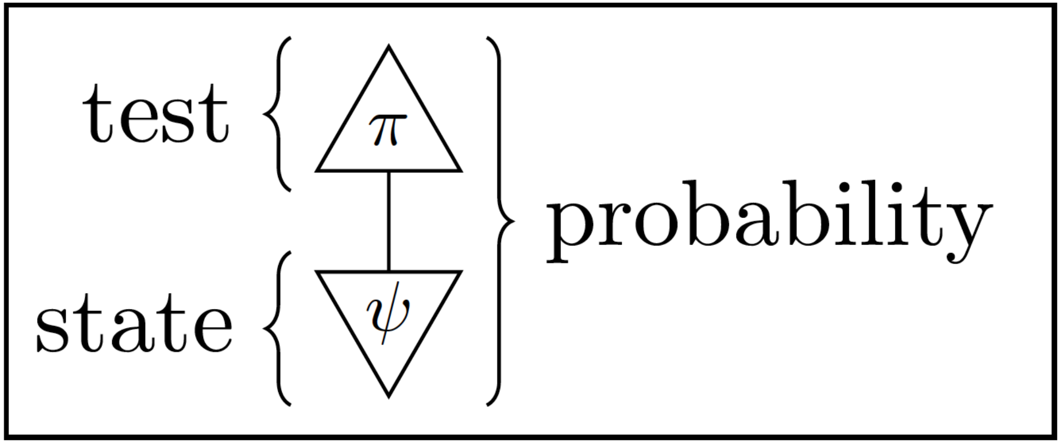
test
$\pi$
state
$\psi$
probability

– *spatio-temporally*/*categorically* –

## *– spatio-temporally/categorically –*

Two operations:

$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

# *– spatio-temporally/categorically –*

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$$\text{“}f \otimes g\text{”} := \text{“}f \ \textbf{while}\ g\text{”}$$

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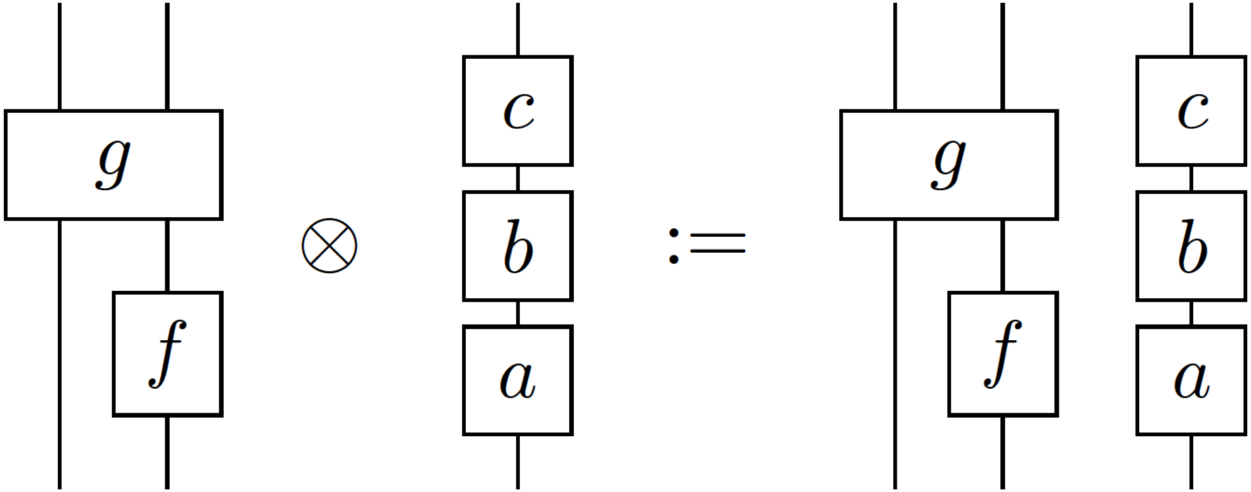

*– spatio-temporally/categorically –*

Two operations:

$$\text{“}f \circ g\text{”} \;:=\; \text{“}f\ \textbf{after}\ g\text{”}$$

---

## – *spatio-temporally/categorically* –

Two operations:

$$\text{“}f \circ g\text{”} := \text{“}f \textbf{ after } g\text{”}$$

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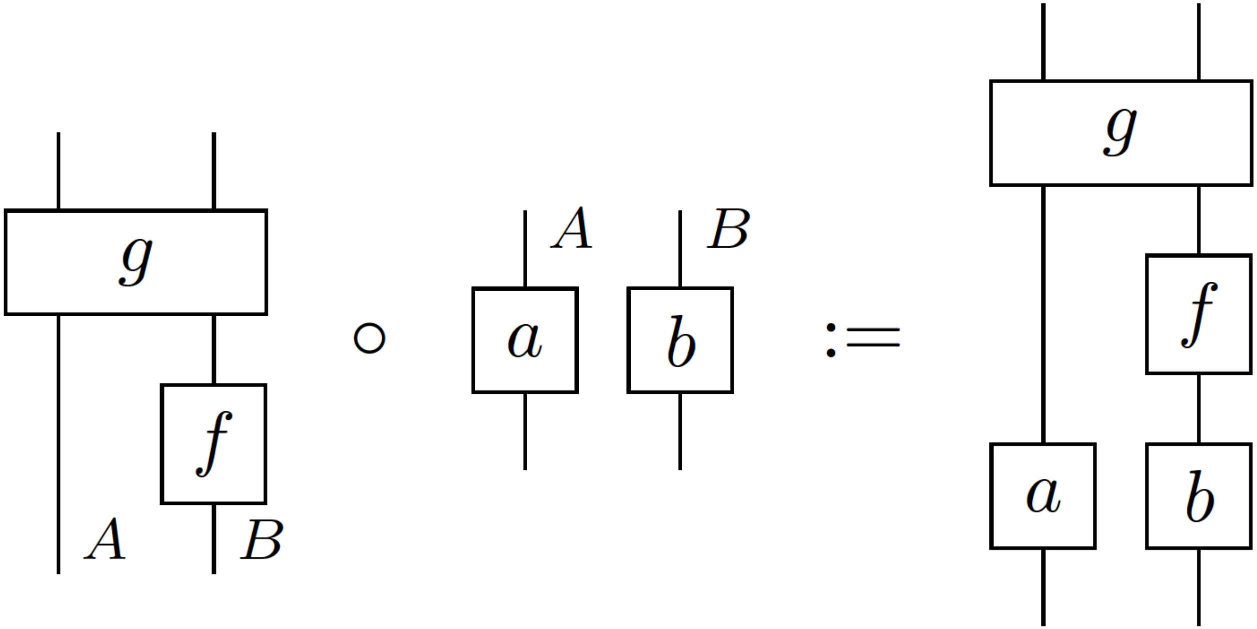


## *– spatio-temporally/categorically –*

Two operations:

$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

$$\text{“}f \circ g\text{”} := \text{“}f \textbf{ after } g\text{”}$$

Formula 1:

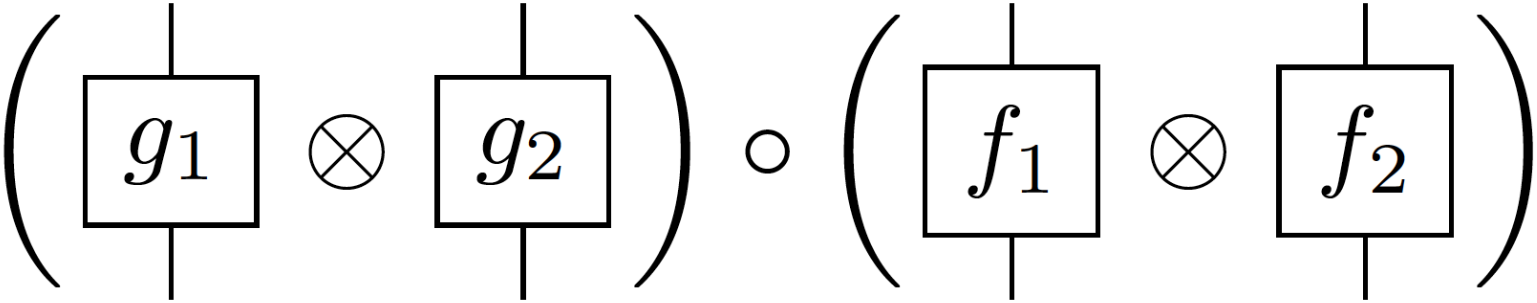
$$(g_1 \otimes g_2) \circ (f_1 \otimes f_2)$$

# *– spatio-temporally/categorically –*

Two operations:

$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

$$\text{“}f \circ g\text{”} := \text{“}f \textbf{ after } g\text{”}$$



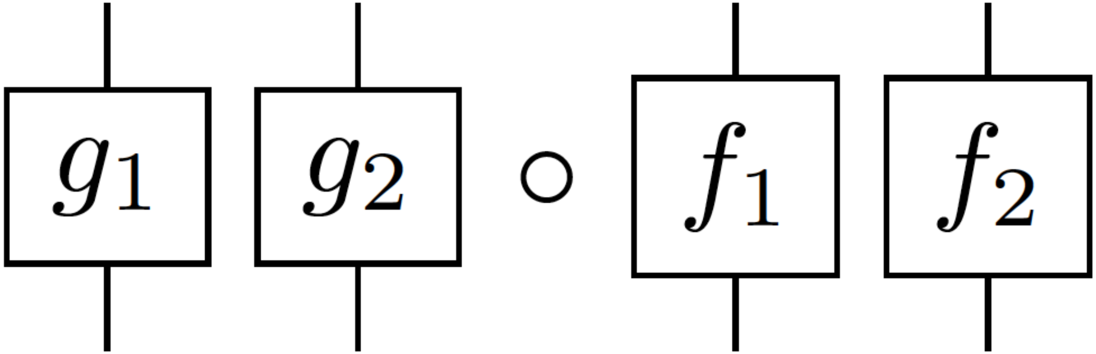
$$\left(\boxed{g_1} \otimes \boxed{g_2}\right) \circ \left(\boxed{f_1} \otimes \boxed{f_2}\right)$$

## – *spatio-temporally*/*categorically* –

Two operations:

“$f \otimes g$” := “$f$ **while** $g$”

“$f \circ g$” := “$f$ **after** $g$”



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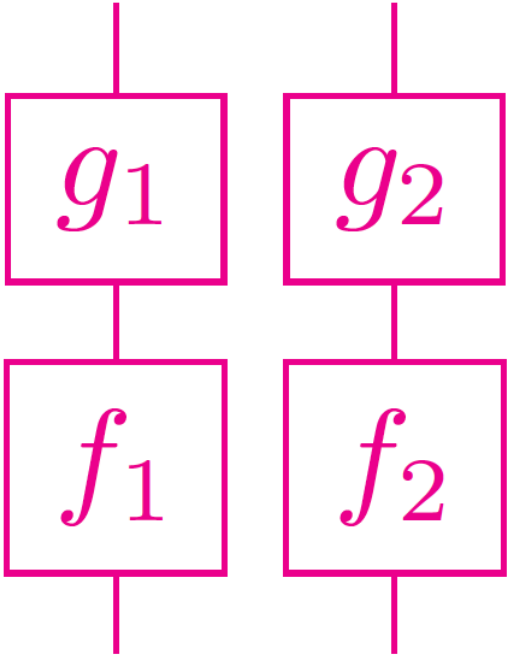
Two operations:

$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

$$\text{“}f \circ g\text{”} := \text{“}f \textbf{ after } g\text{”}$$



## *– spatio-temporally/categorically –*

Two operations:

$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

$$\text{“}f \circ g\text{”} := \text{“}f \textbf{ after } g\text{”}$$

Formula 2:

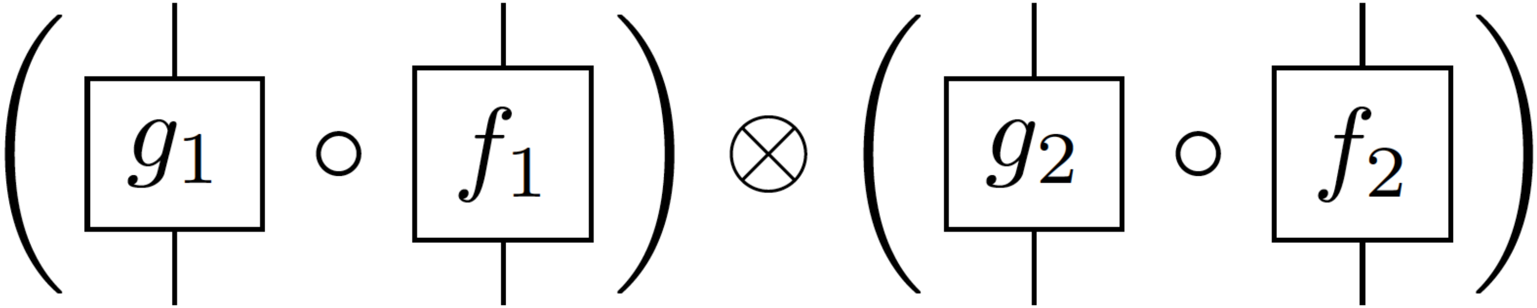
$$(g_1 \circ f_1) \otimes (g_2 \circ f_2)$$

## *– spatio-temporally/categorically –*

Two operations:

$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

$$\text{“}f \circ g\text{”} := \text{“}f \textbf{ after } g\text{”}$$



*– spatio-temporally/categorically –*

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$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

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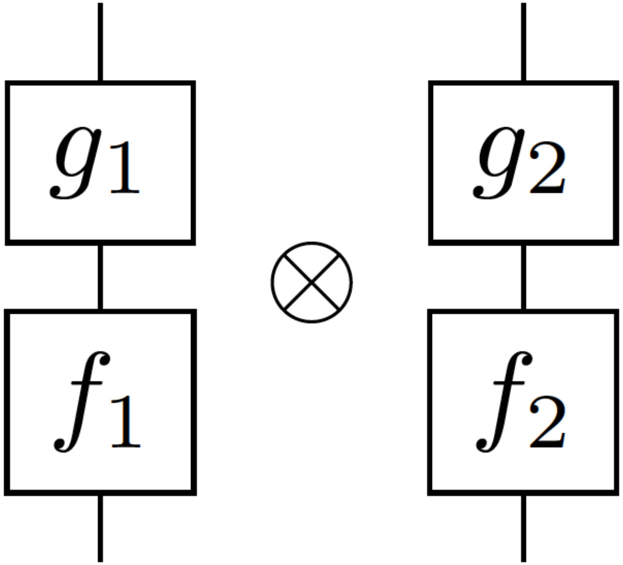
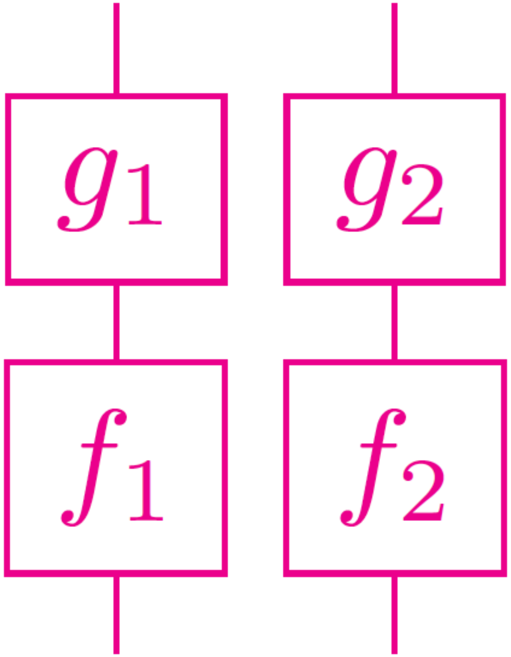


# *– spatio-temporally/categorically –*

Two operations:

$$\text{“}f \otimes g\text{”} := \text{“}f \textbf{ while } g\text{”}$$

$$\text{“}f \circ g\text{”} := \text{“}f \textbf{ after } g\text{”}$$

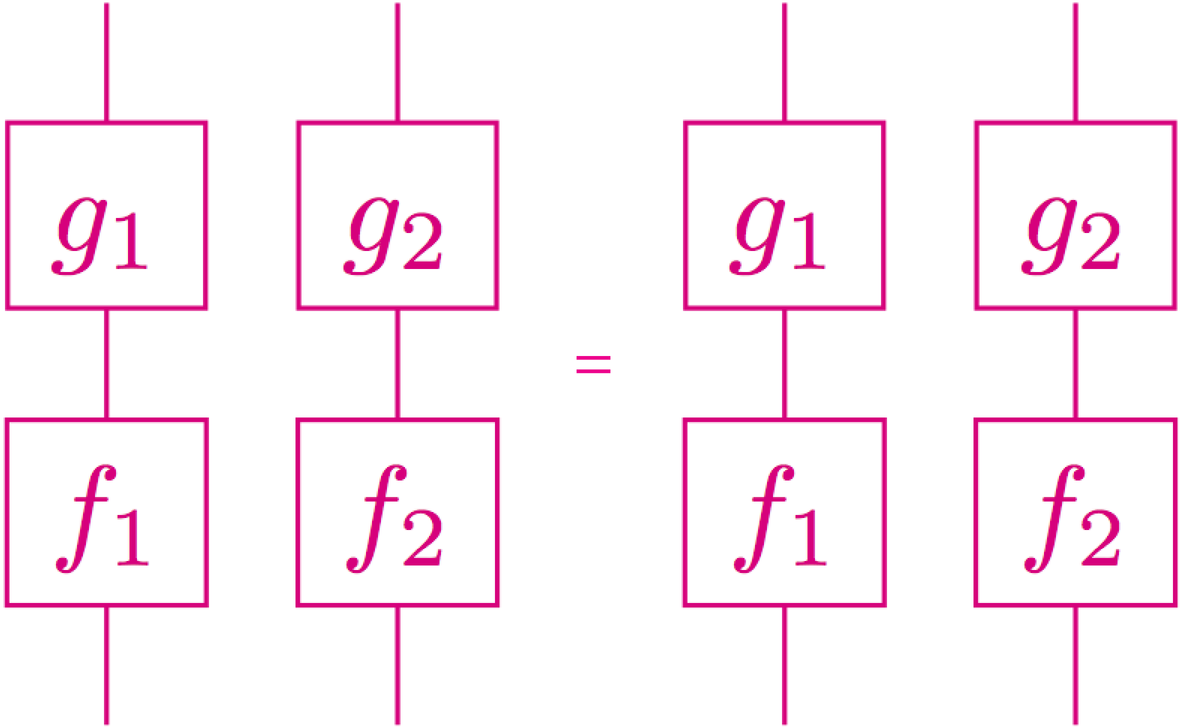


– *spatio-temporally*/*categorically* –

Algebra:

$$(g_1 \otimes g_2) \circ (f_1 \otimes f_2) \; = \; (g_1 \circ f_1) \otimes (g_2 \circ f_2)$$

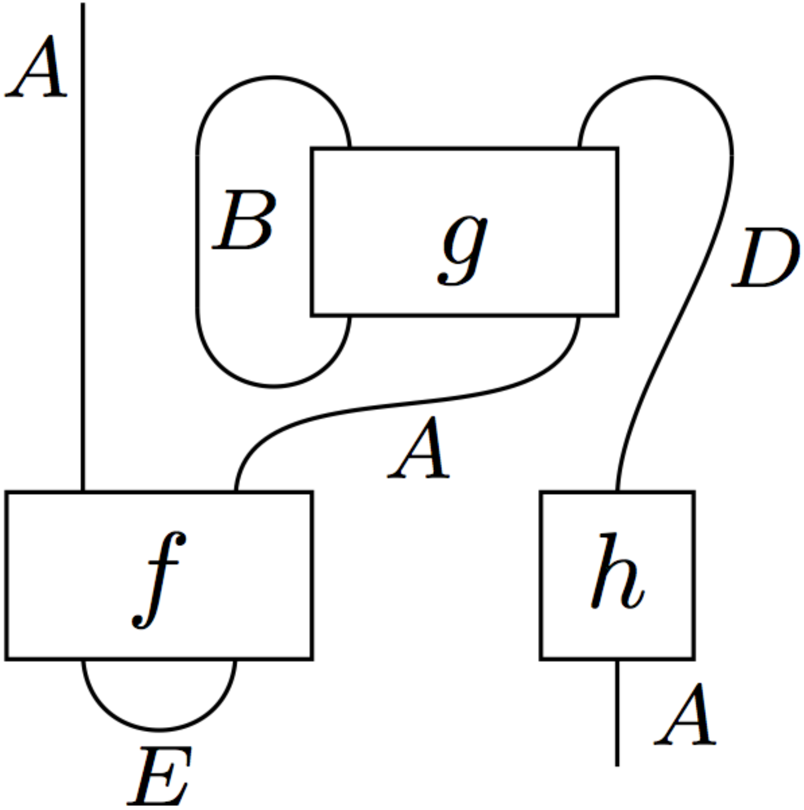
Diagrams:



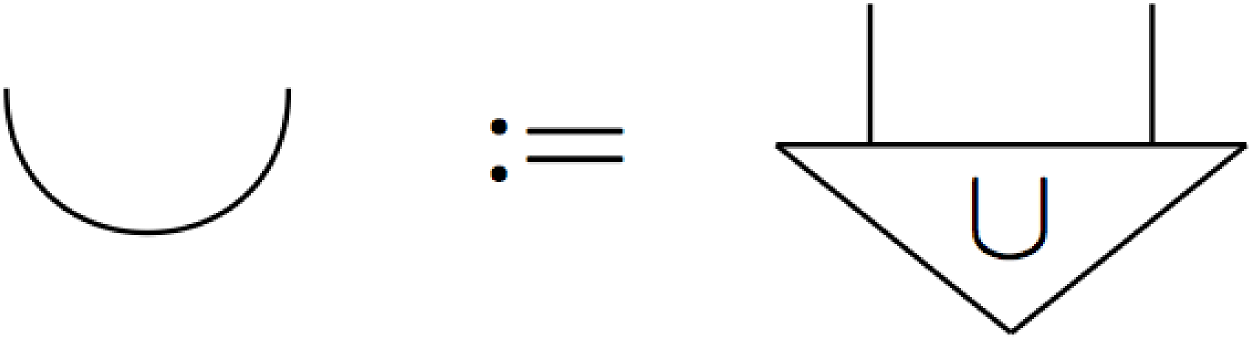
# quantum wires and boxes

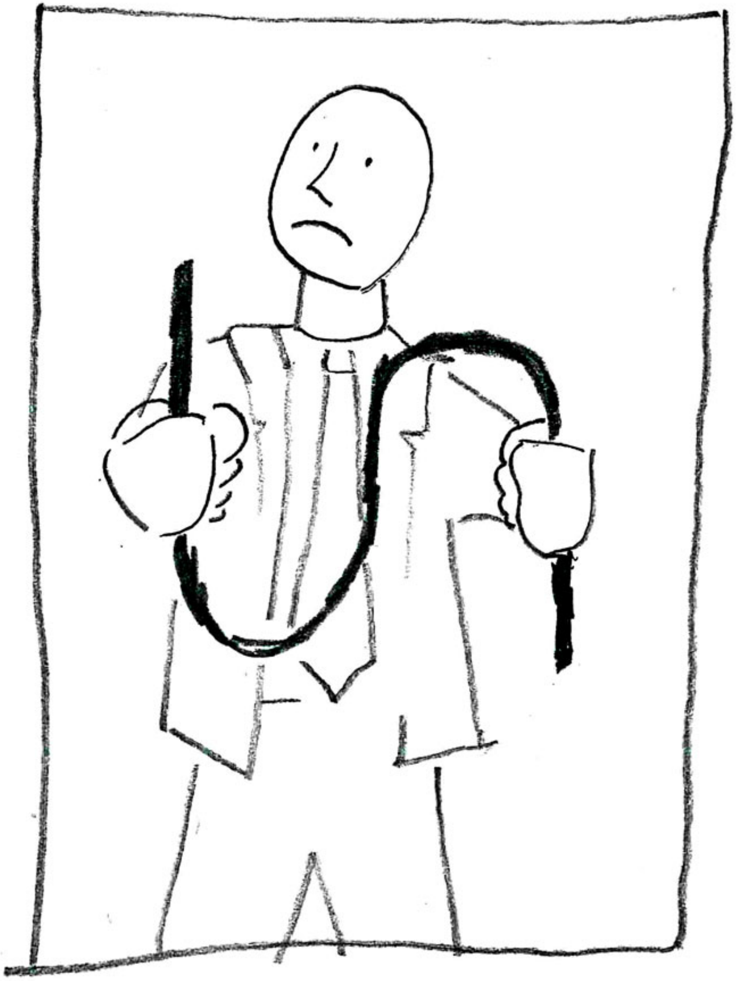
*When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own.* ***I would not call that one but rather the characteristic trait of quantum mechanics****, the one that enforces its entire departure from classical lines of thought.*

— Erwin Schrödinger, 1935.

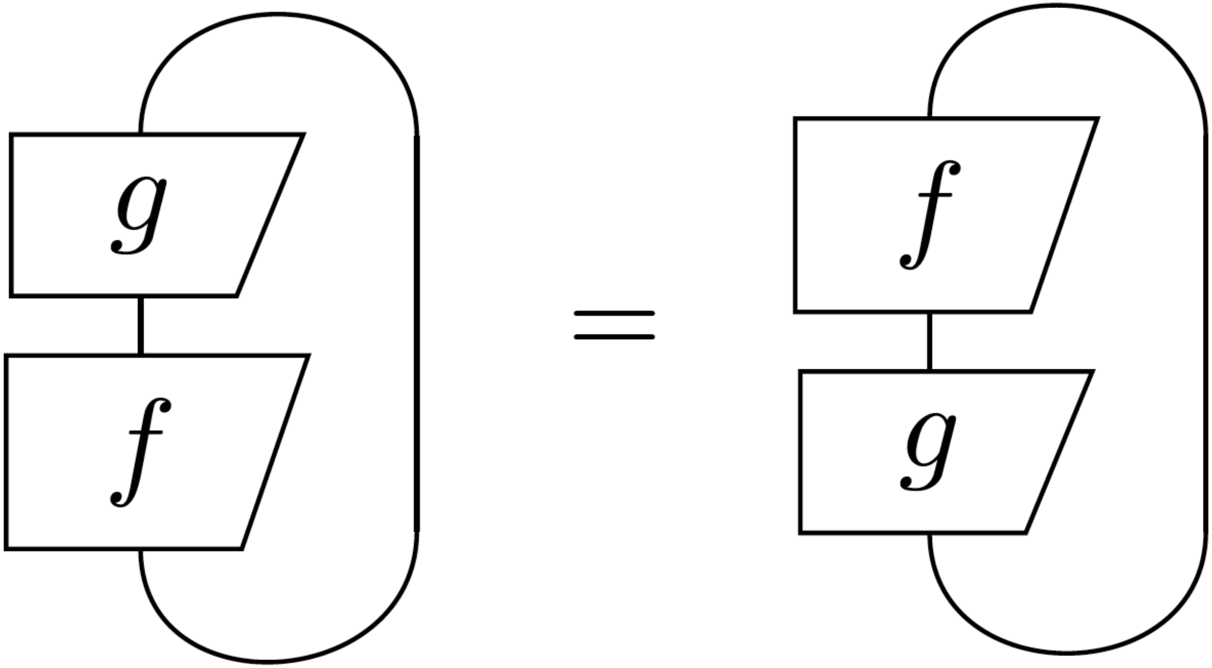

A
B
g
D
A
f
h
A
E

# *– entanglement –*

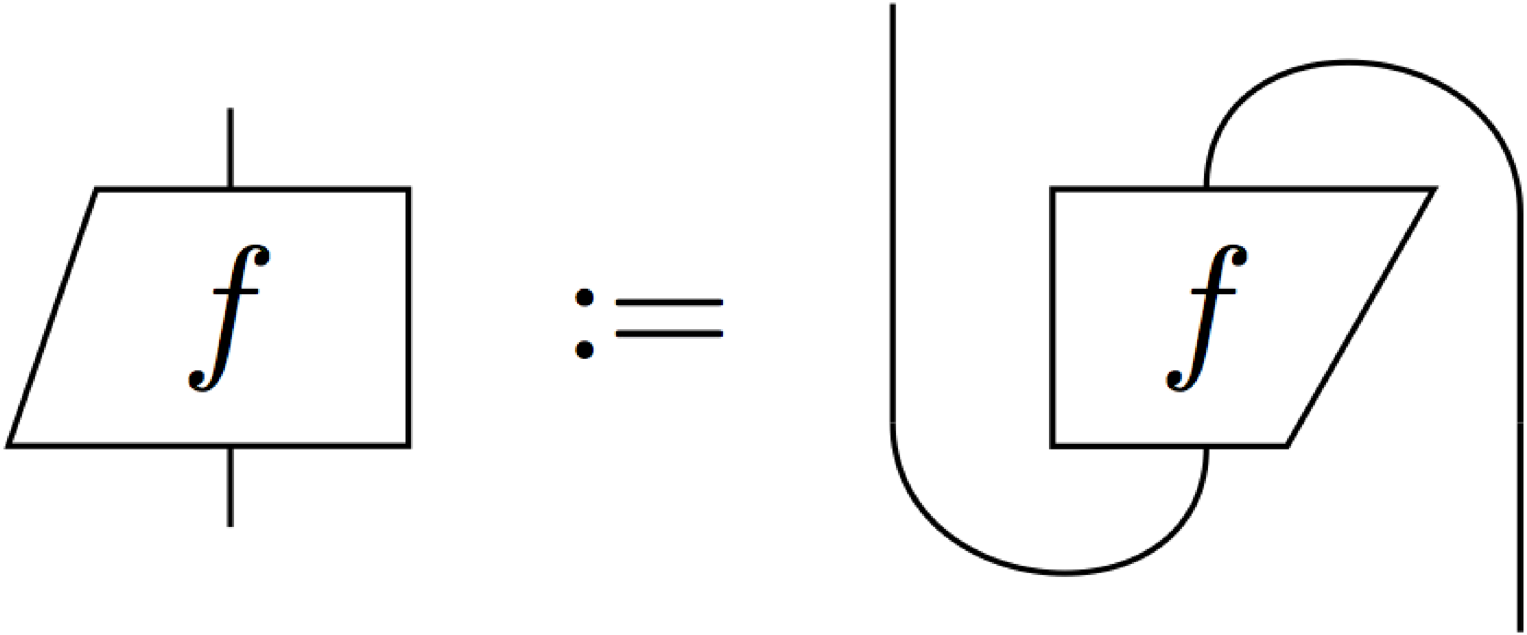




# – *trace* –



# – transpose –



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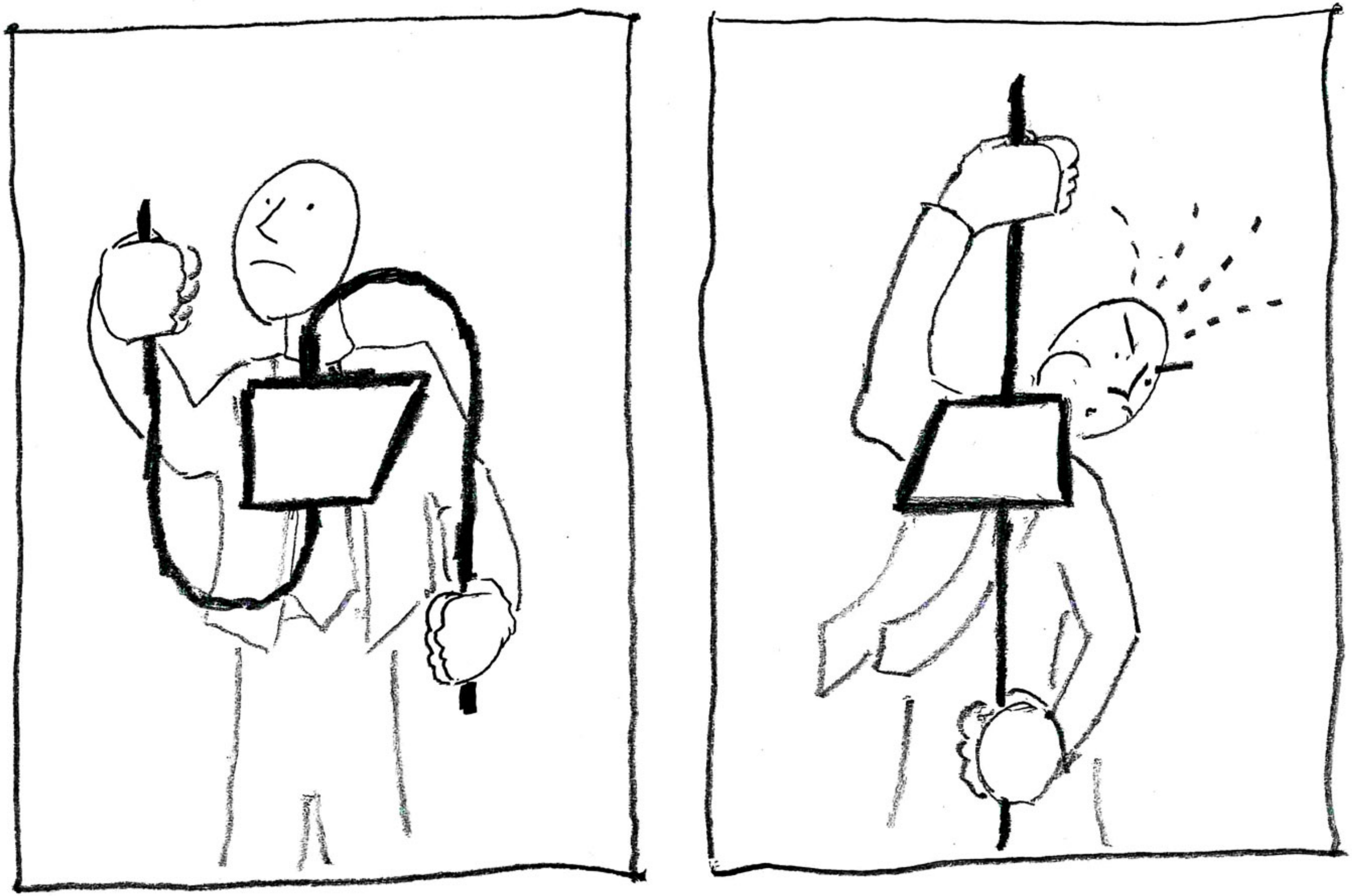
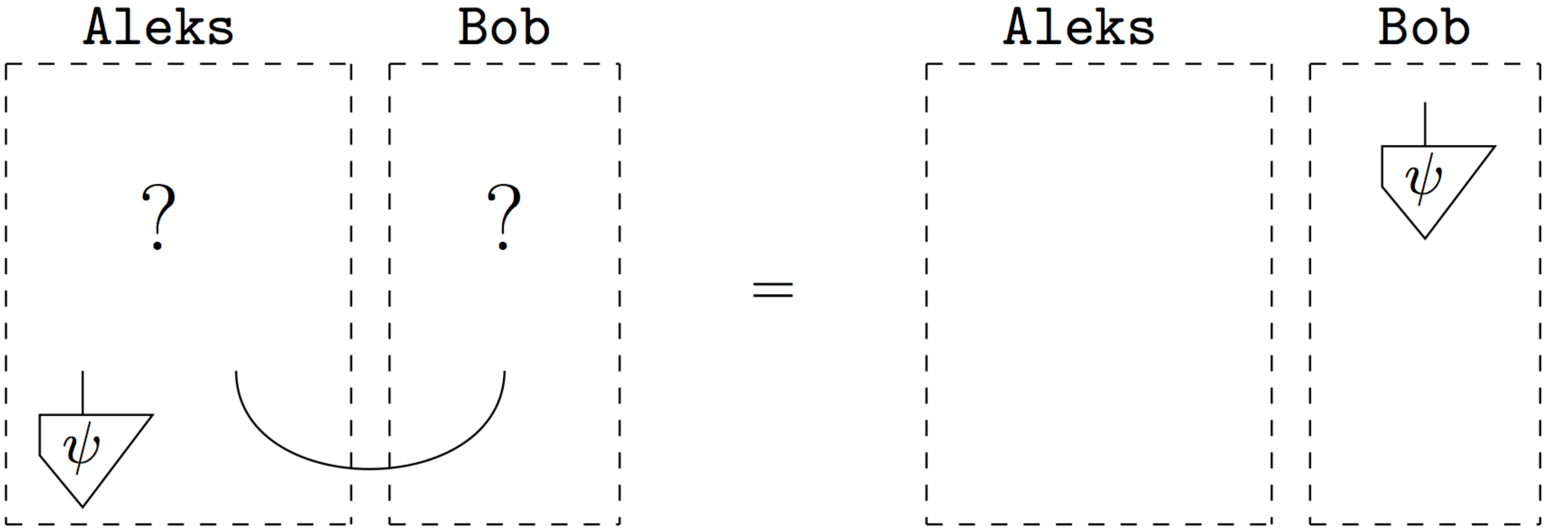
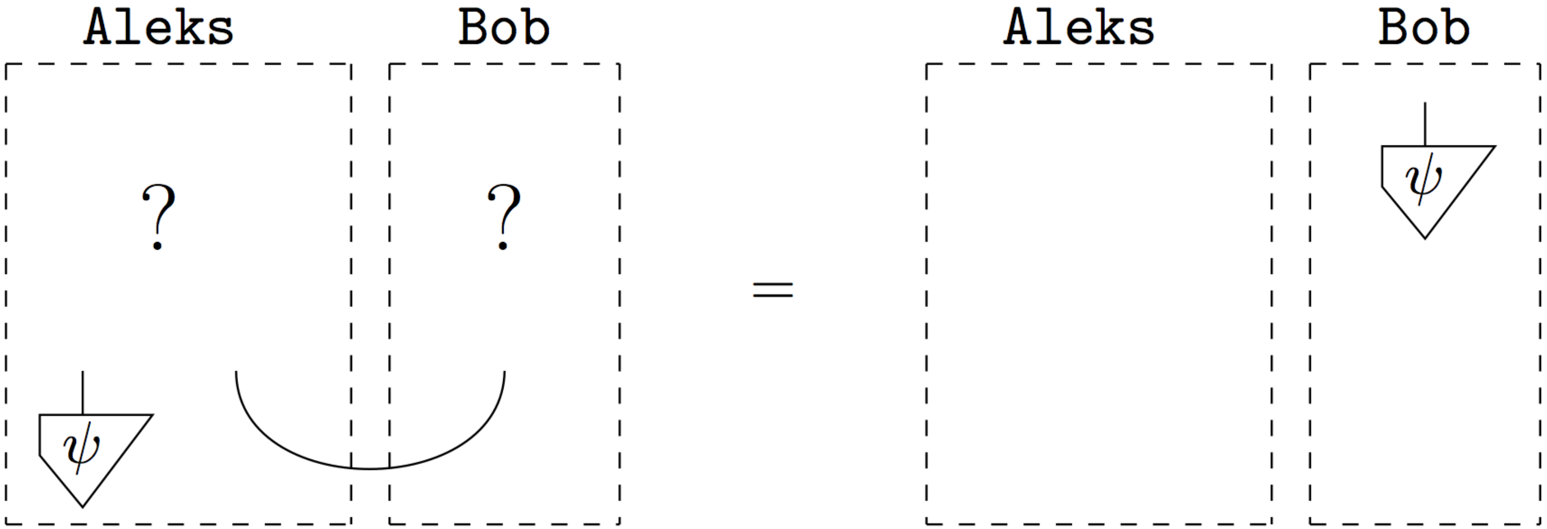


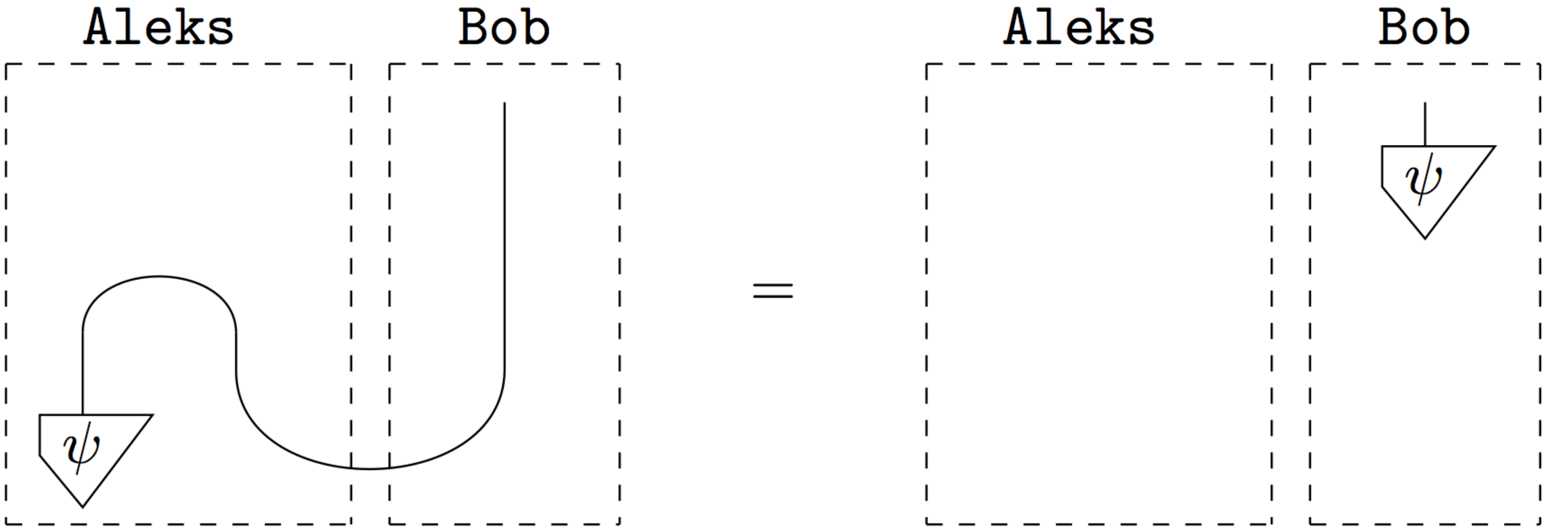
# *– quantum teleportation –*



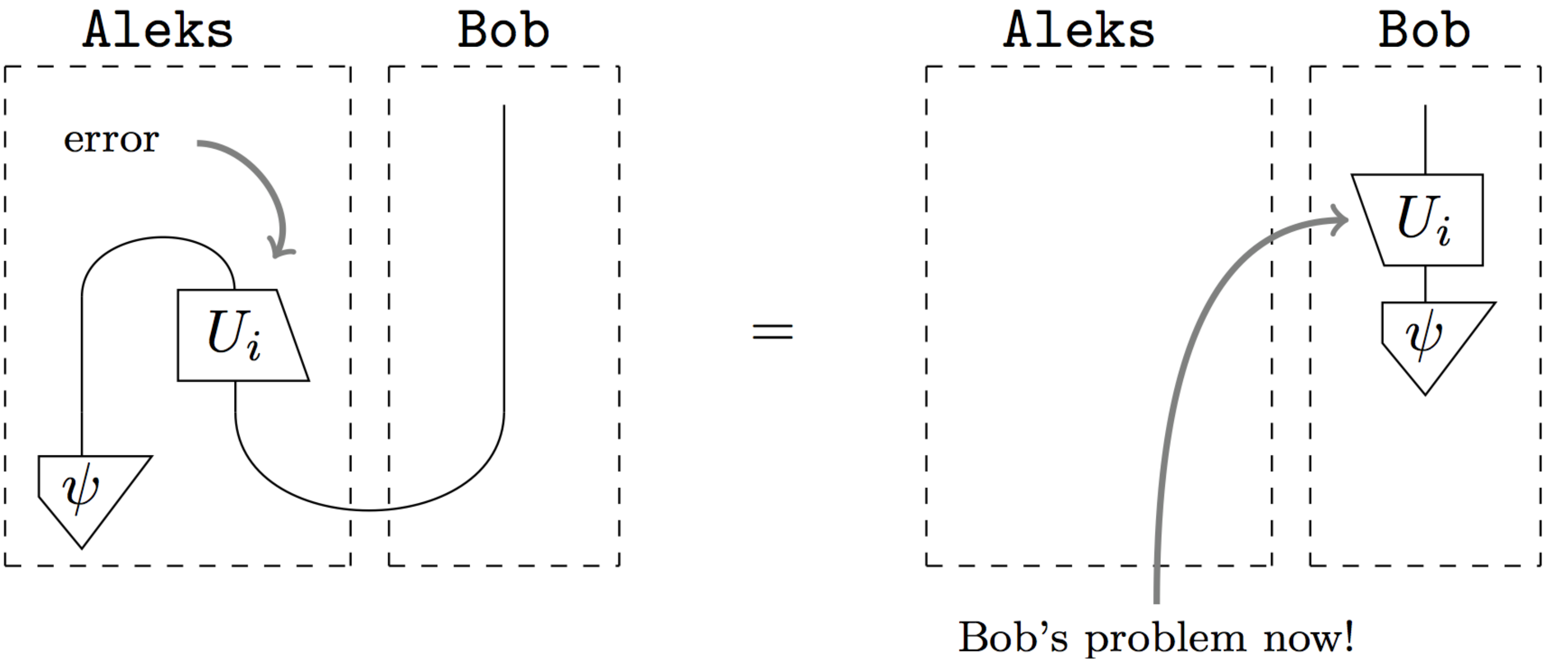
# – *quantum teleportation* –



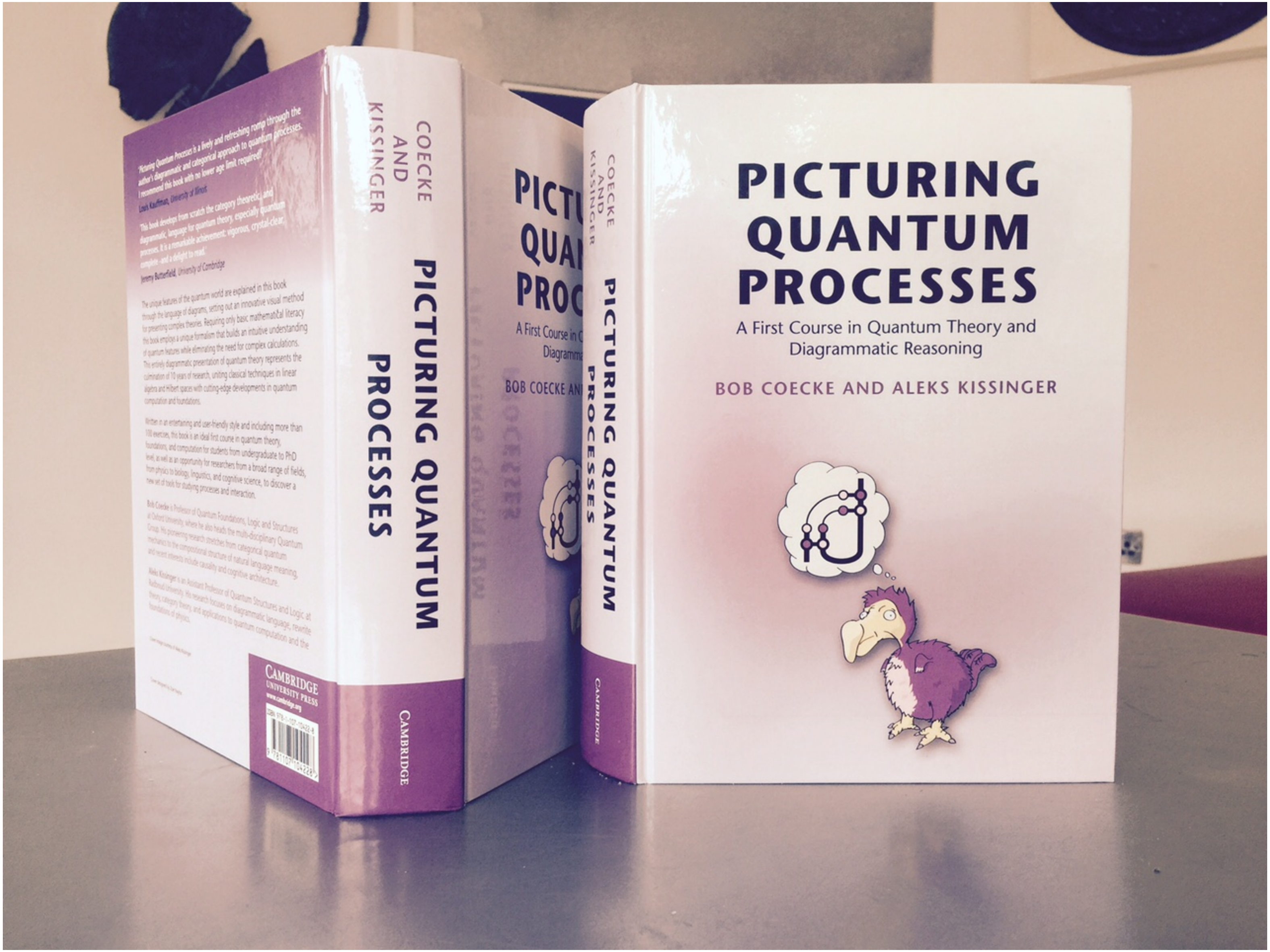
# – quantum teleportation –



# – quantum teleportation –



B. Coecke (2005) *Kindergarten Quantum Mechanics*. arXiv:quant-ph/0510032


PICTURING QUANTUM PROCESSES
A First Course in Quantum Theory and Diagrammatic Reasoning
BOB COECKE AND ALEKS KISSINGER
COECKE AND KISSINGER
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www.cambridge.org

'Picturing Quantum Processes is a lively and refreshing romp through the author's diagrammatic and categorical approach to quantum processes. I recommend this book with no lower age limit required!'

Louis Kauffman, *University of Illinois*

'This book develops from scratch the category theoretic, and diagrammatic, language for quantum theory, especially quantum processes. It is a remarkable achievement: vigorous, crystal-clear, complete - and a delight to read.'

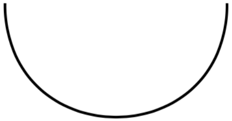
Jeremy Butterfield, *University of Cambridge*

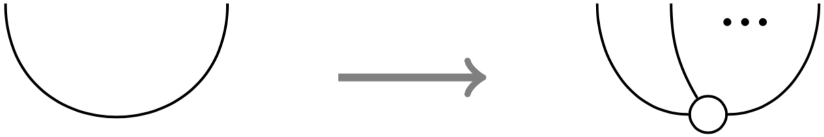
The unique features of the quantum world are explained in this book through the language of diagrams, setting out an innovative visual method for presenting complex theories. Requiring only basic mathematical literacy this book employs a unique formalism that builds an intuitive understanding of quantum features while eliminating the need for complex calculations. This entirely diagrammatic presentation of quantum theory represents the culmination of 10 years of research, uniting classical techniques in linear algebra and Hilbert spaces with cutting-edge developments in quantum computation and foundations.

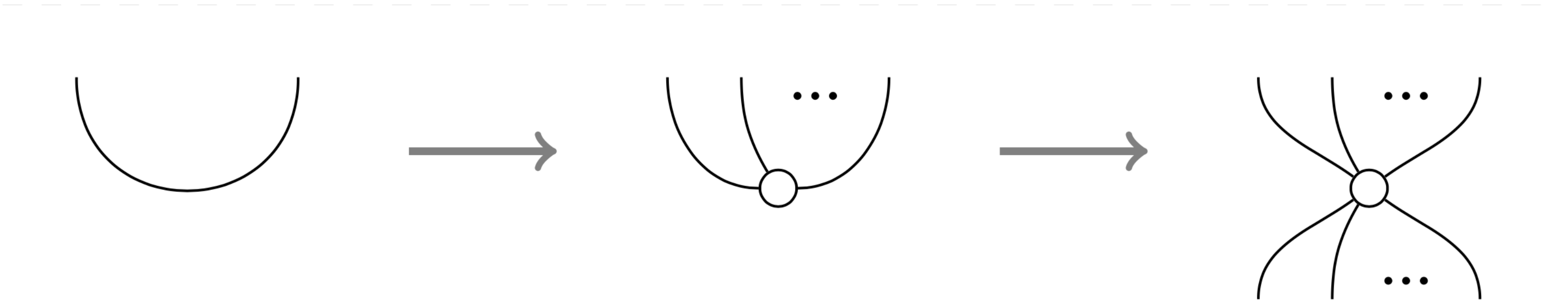
Written in an entertaining and user-friendly style and including more than 100 exercises, this book is an ideal first course in quantum theory, foundations, and computation for students from undergraduate to PhD level, as well as an opportunity for researchers from a broad range of fields, from physics to biology, linguistics, and cognitive science, to discover a new set of tools for studying processes and interaction.

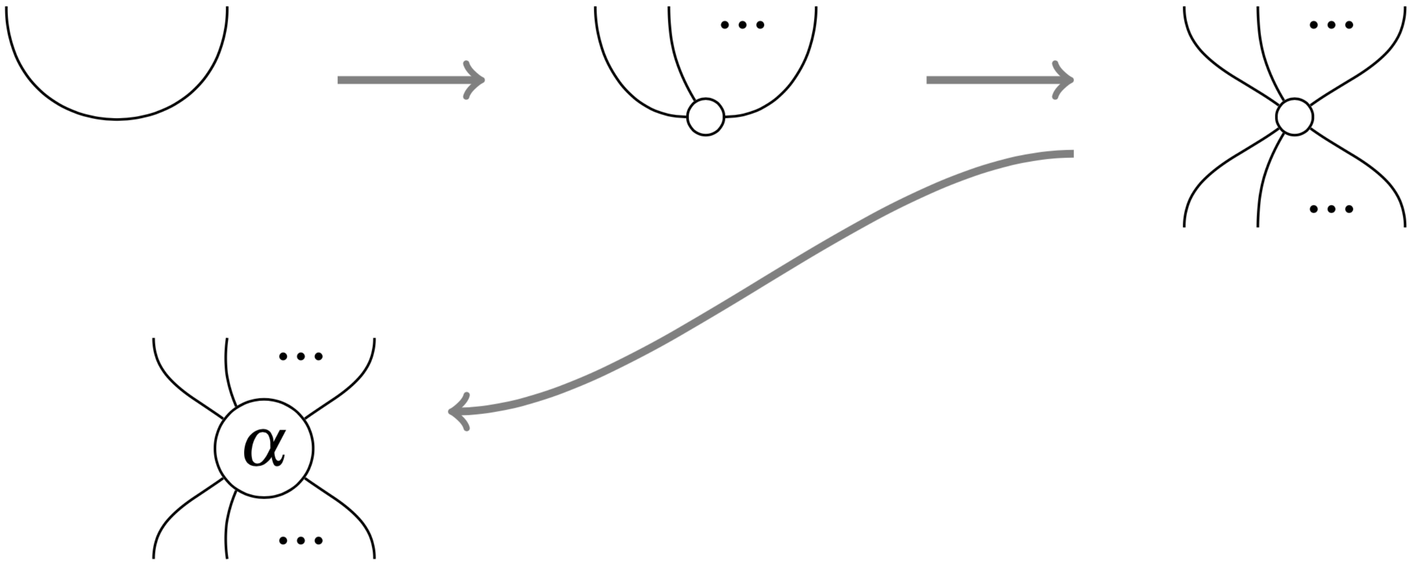
**Bob Coecke** is Professor of Quantum Foundations, Logic and Structures at Oxford University, where he also heads the multi-disciplinary Quantum Group. His pioneering research stretches from categorical quantum mechanics to the compositional structure of natural language meaning, and recent interests include causality and cognitive architecture.

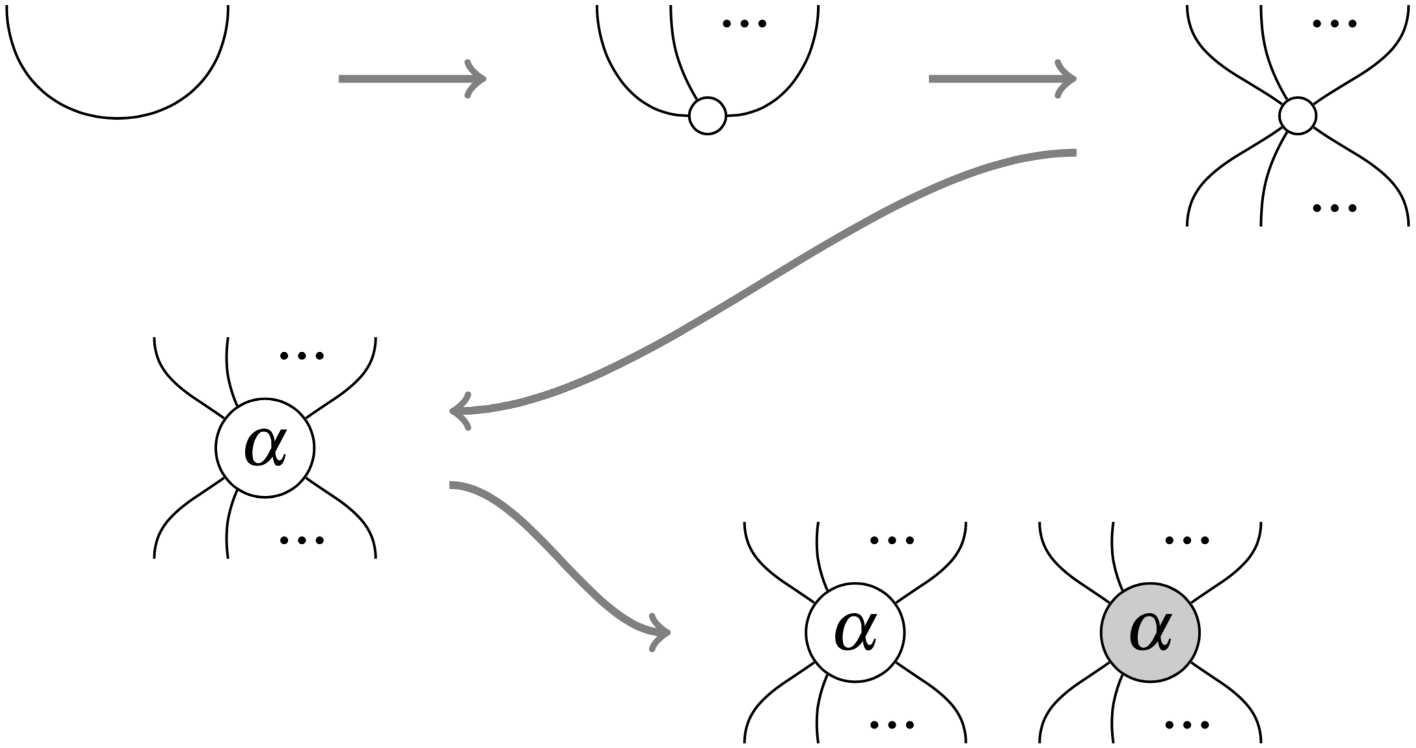
**Aleks Kissinger** is an Assistant Professor of Quantum Structures and Logic at Radboud University. His research focuses on diagrammatic language, rewrite theory, category theory, and applications to quantum computation and the foundations of physics.



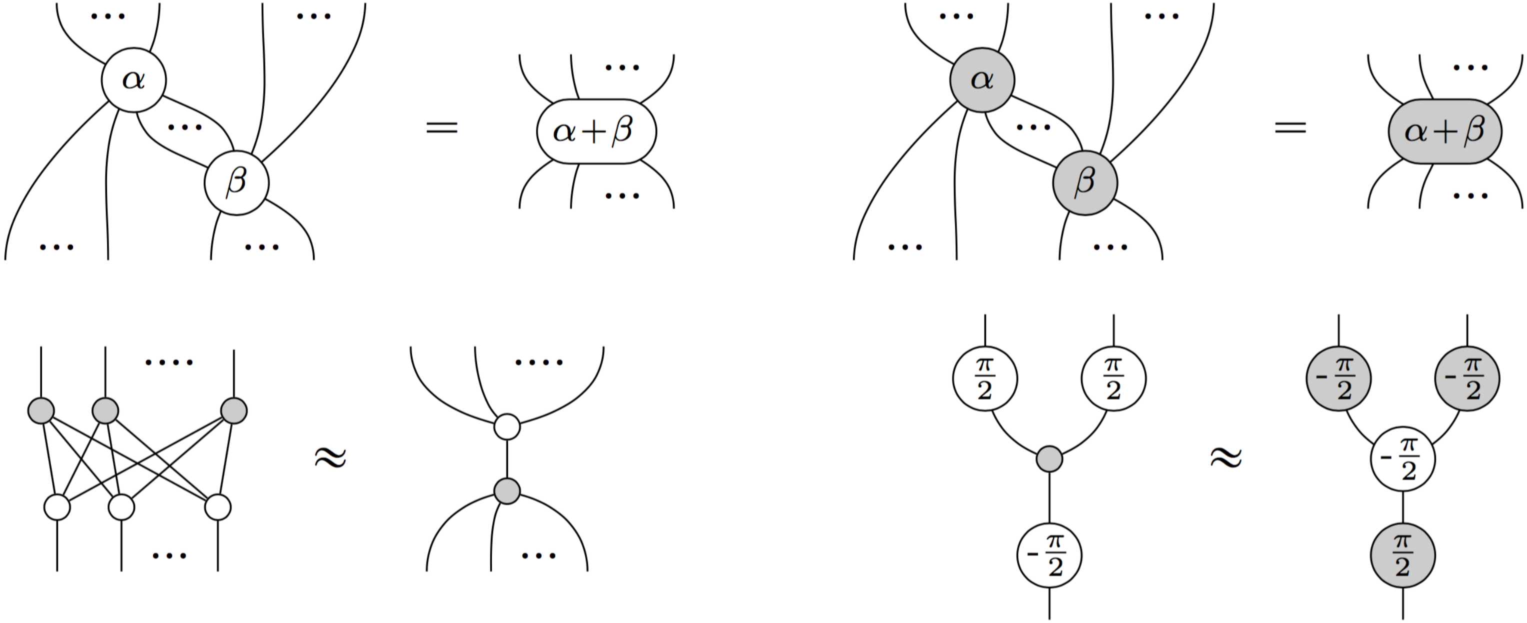





α

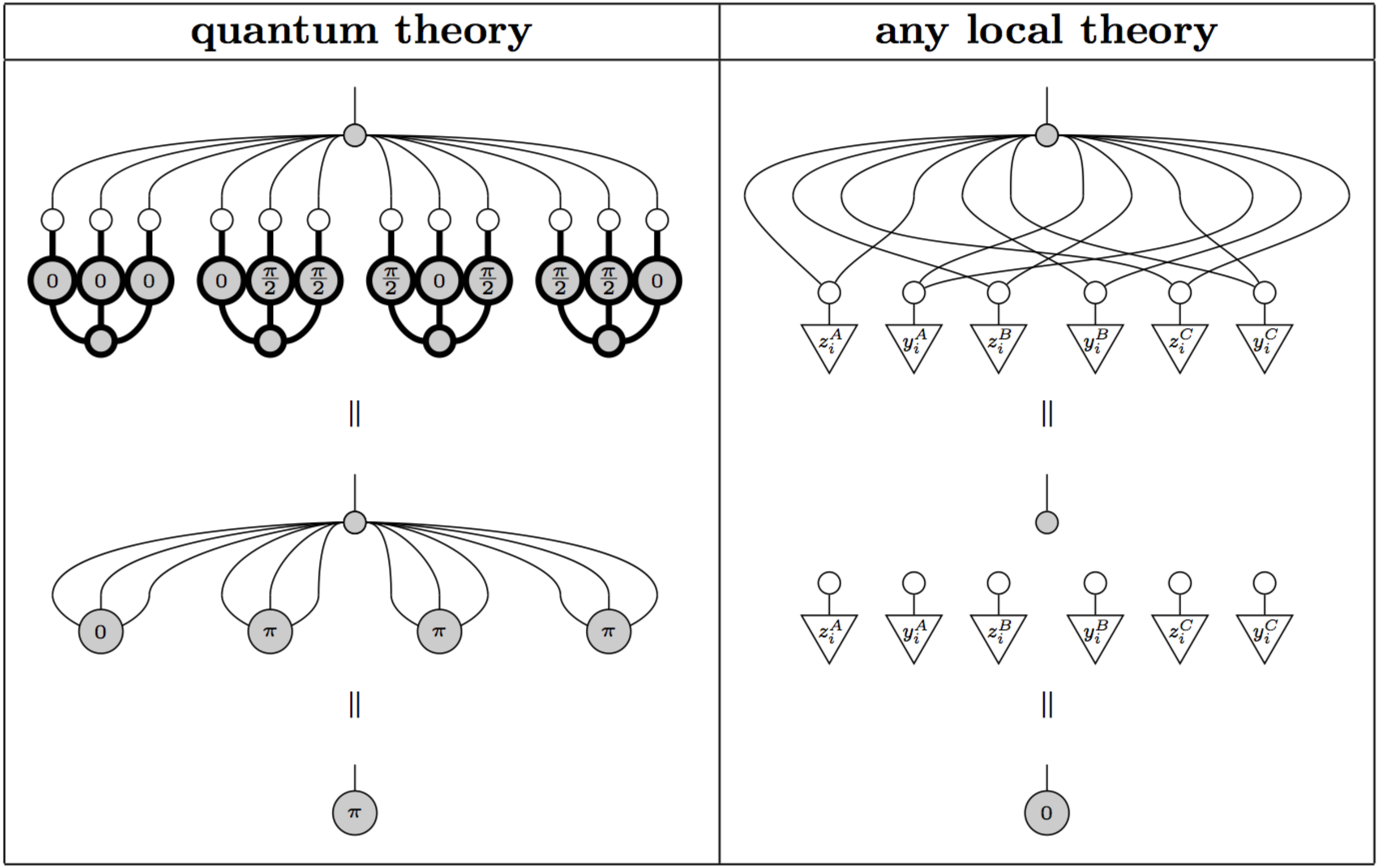

α
α
α

# – stabiliser ZX-calculus –



B. Coecke & Ross Duncan (ICALP 2008) *Interacting quantum observables: categorical algebra and diagrammatics.* NJP’11. arXiv:quant-ph/09064725

# *– quantum non-locality –*



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B. Coecke, Duncan, Kissinger & Quanlong Wang (LiCS 2012) *Strong complementarity and non-locality in categorical quantum mechanics*. arXiv:1203.4988

# *– completeness –*

Peter Selinger (2008) Any equational statement is provable for string diagrams if and only if it is provable for Hilbert spaces and linear maps.

Miriam Backens (2012) ... stabiliser restriction of **ZX-calculus**...

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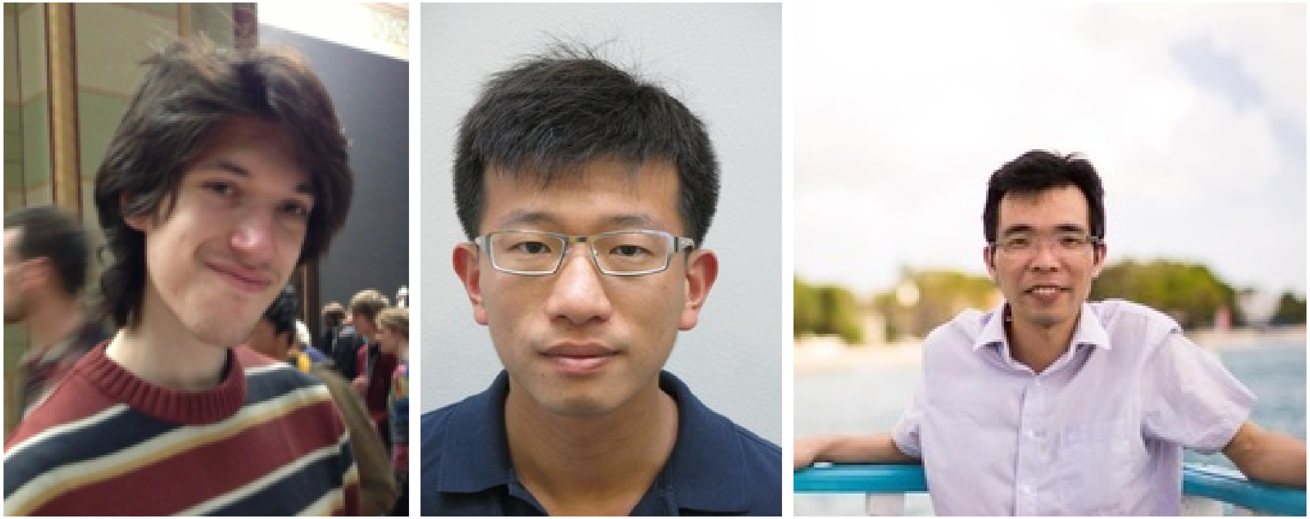

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Kang Feng Ng and Quanlong Wang (2017) ... everything...

E. Jeandel, S. Perdrix & R. Vilmart (2018) ... everything, better...

Kang Feng Ng and Quanlong Wang (2018) ... everything, even better...

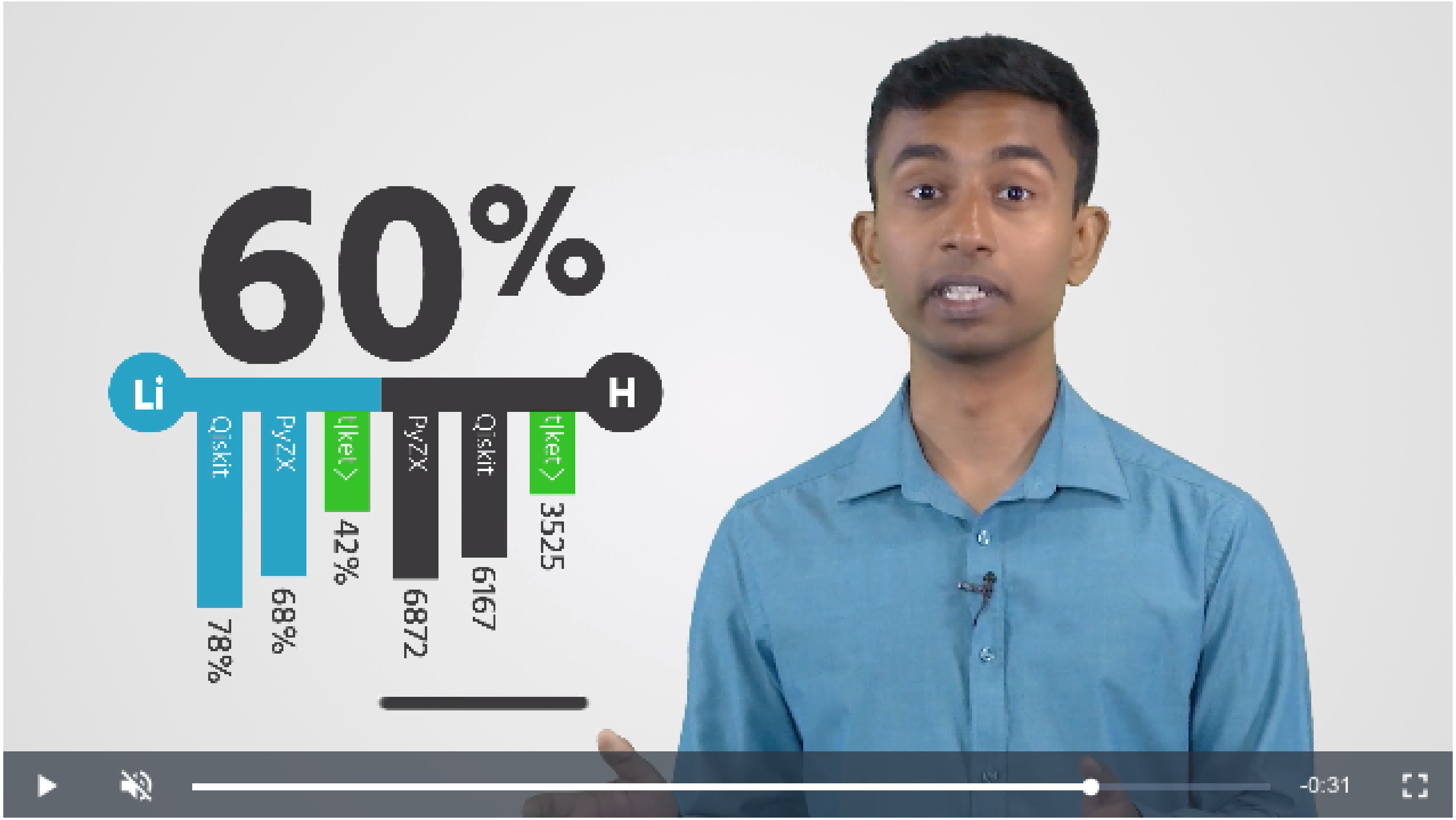
Bob Coecke and Quanlong Wang (2018) ... 2 qubits only, but way better...

Renaud Vilmart (2019) ... everything, way better...

Kang Feng Ng and Quanlong Wang (17.9 mins) ... everything, way way better...

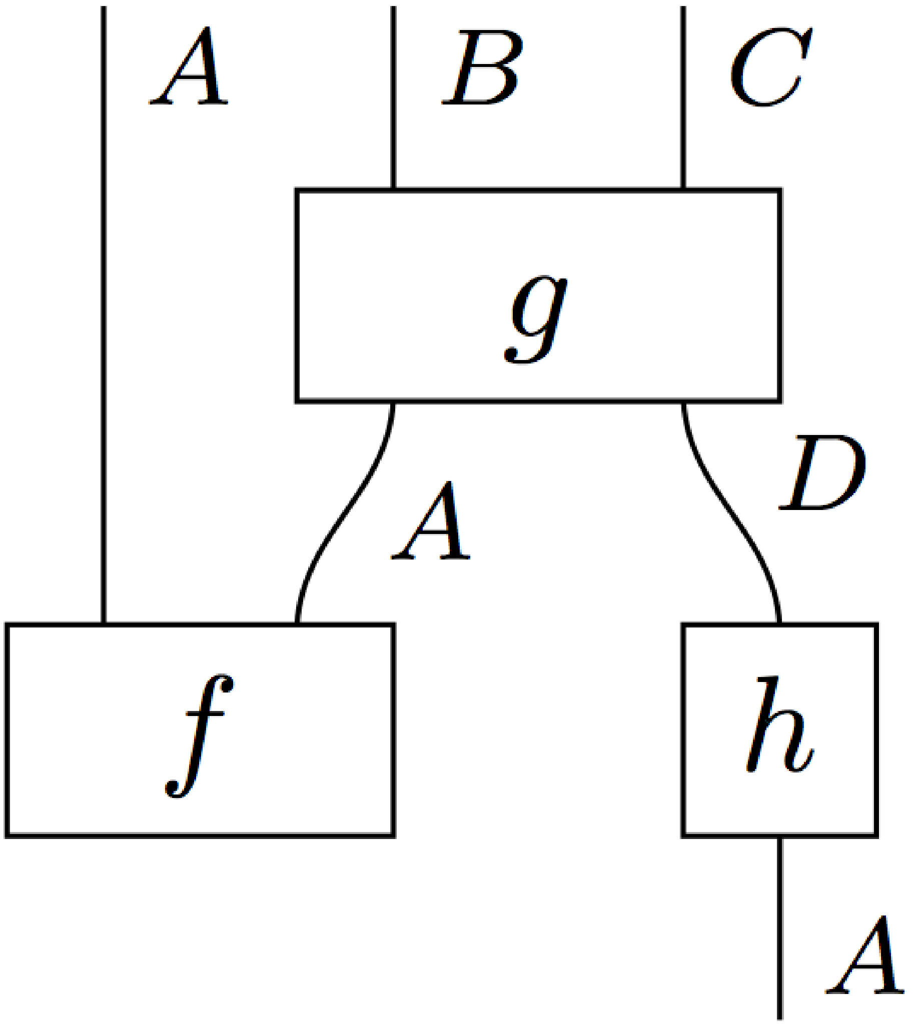
E. Jeandel, S. Perdrix & R. Vilmart (3.4 secs ago) ... everything, way$^3$ better...

# *– ZX-calculus now outperforms anything else!–*



# causal wires and boxes

**Diagram** :=



# Process theory := interpretation of diagrams

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- assign to each wire a **system**,
- assign to each box a **process**,
- **interpret 'wiring together'**

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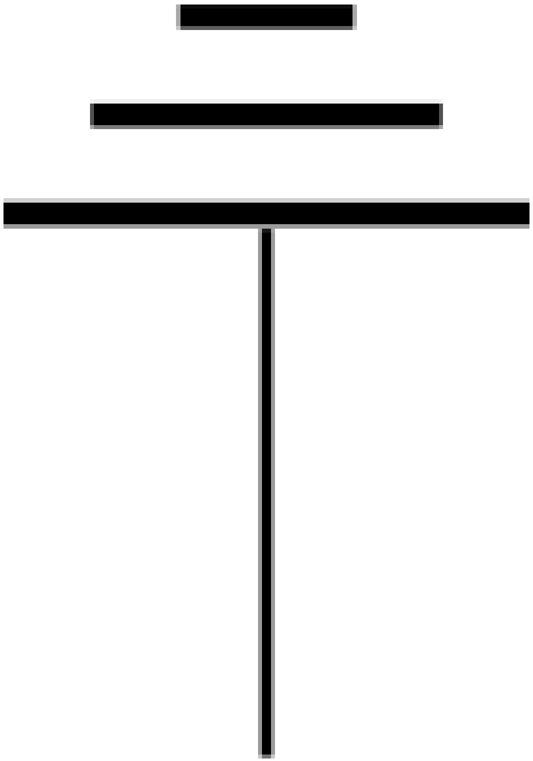
  - as causal influences

# Discarding effect:



# Causality for processess :=

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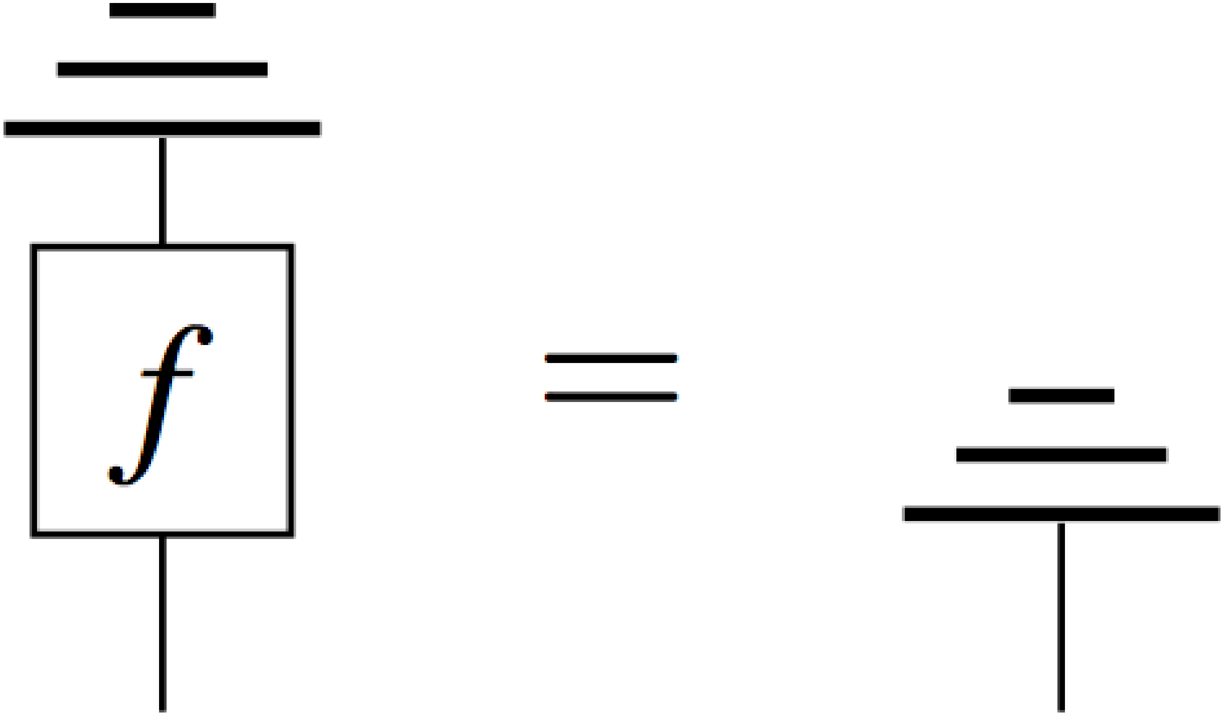

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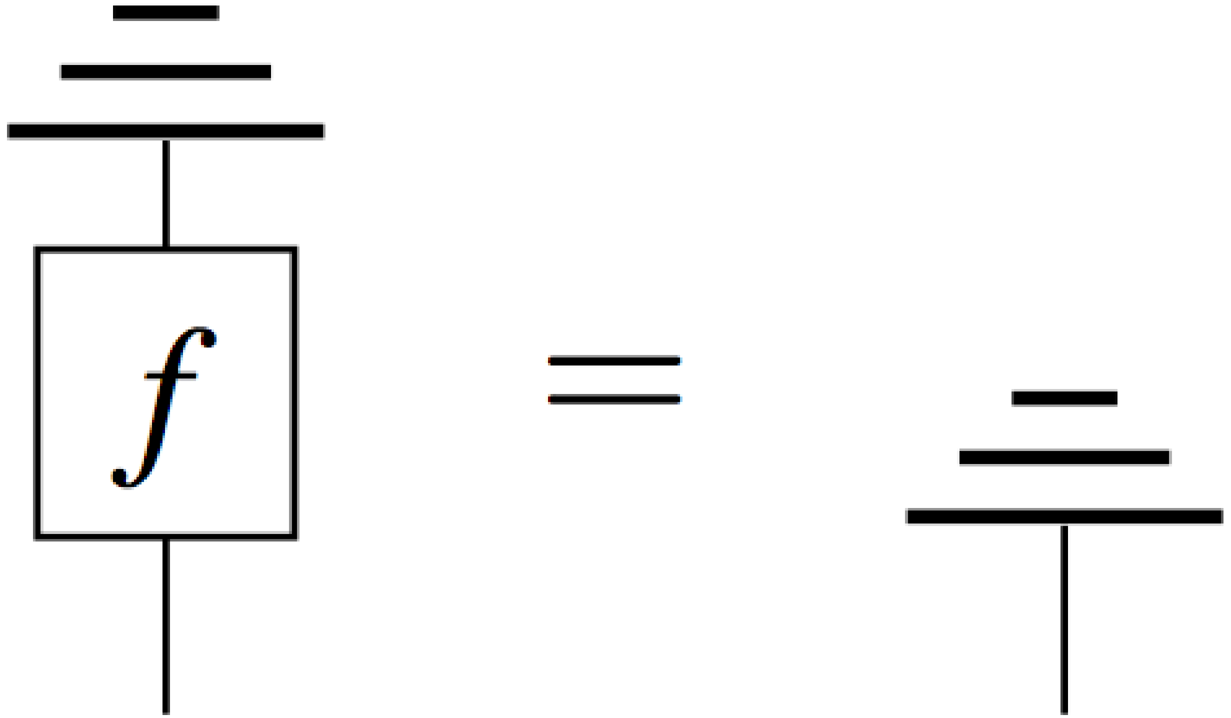
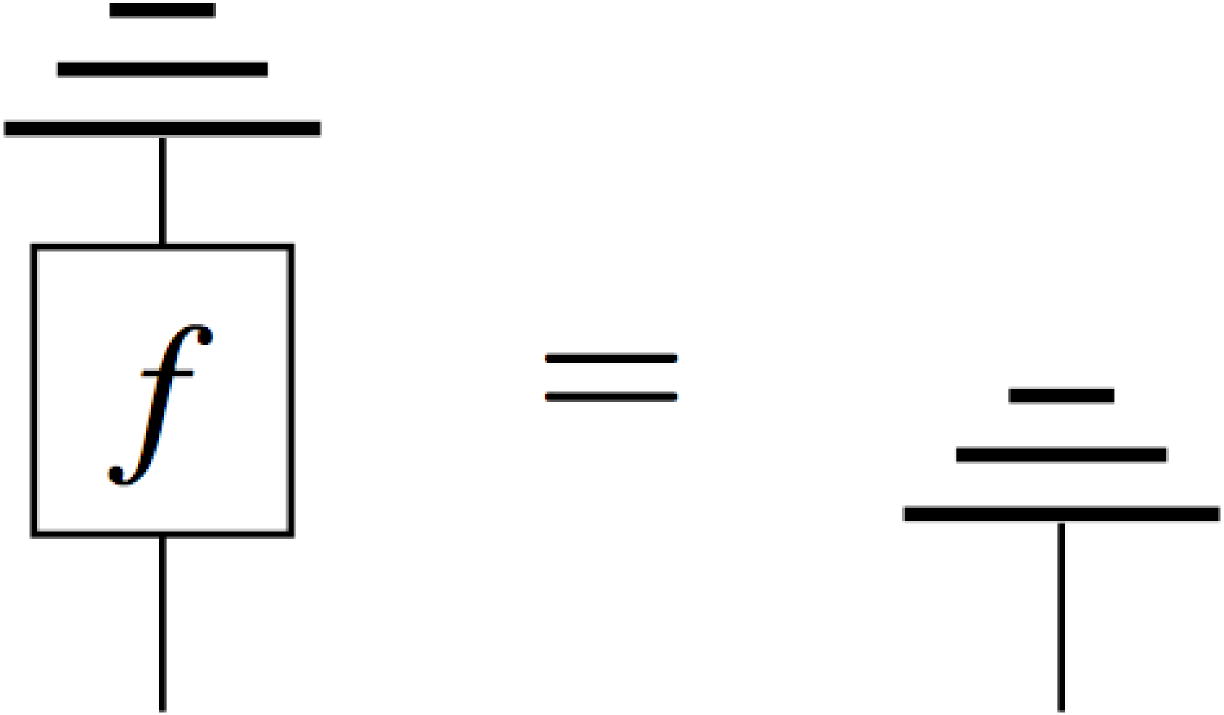


*When the output of a process is discarded, the process itself may also be discarded.*

**Causality** for processess :=



*A necessity for doing science!*

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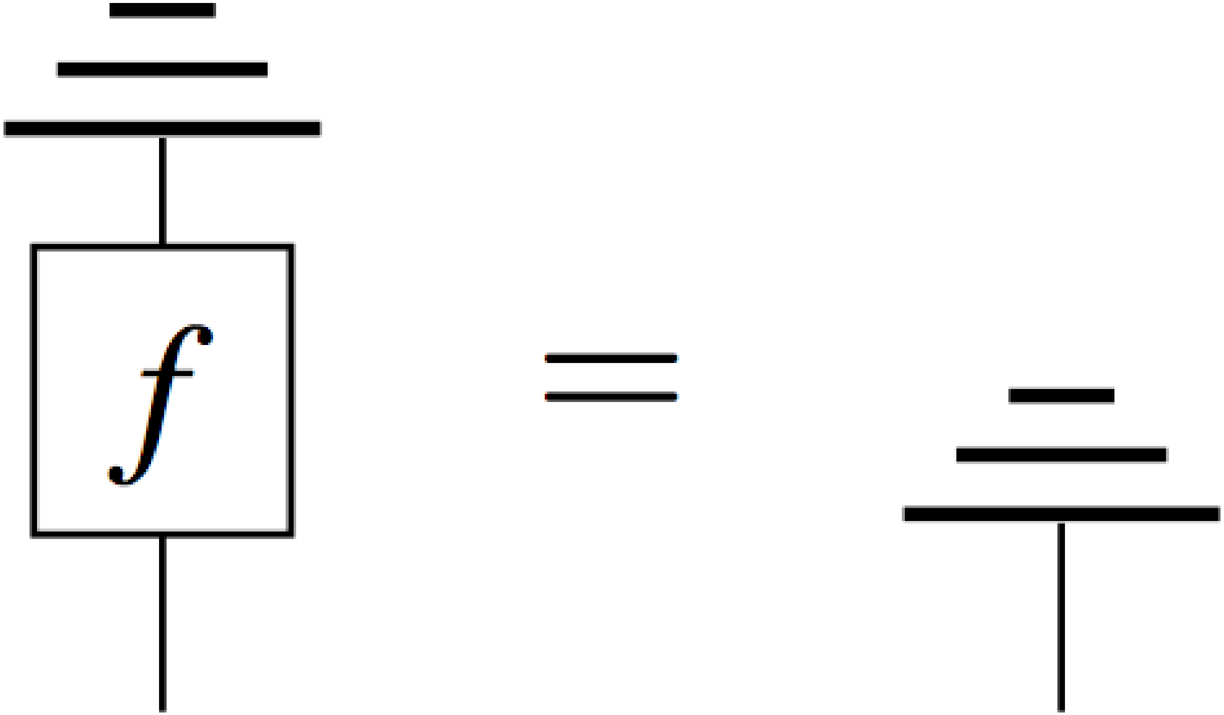
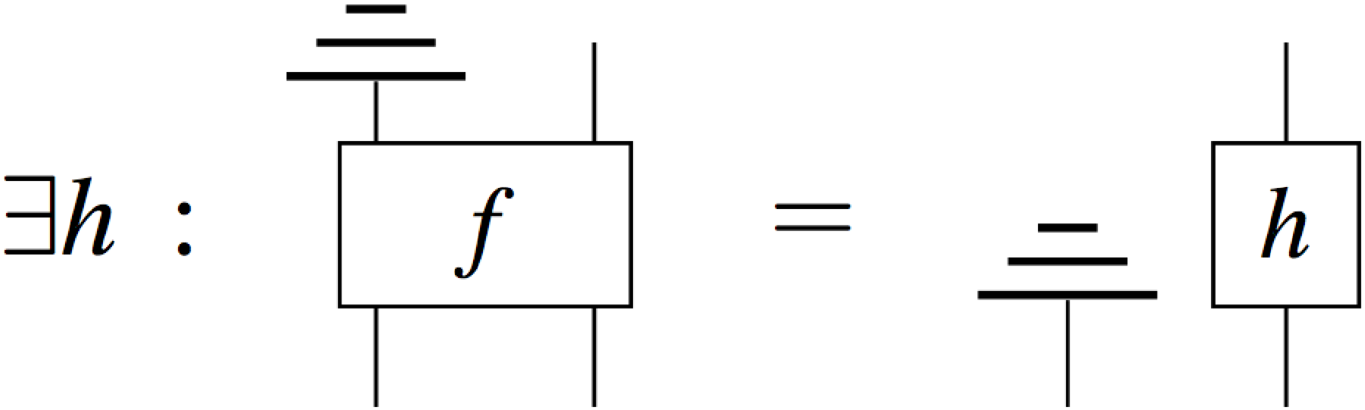


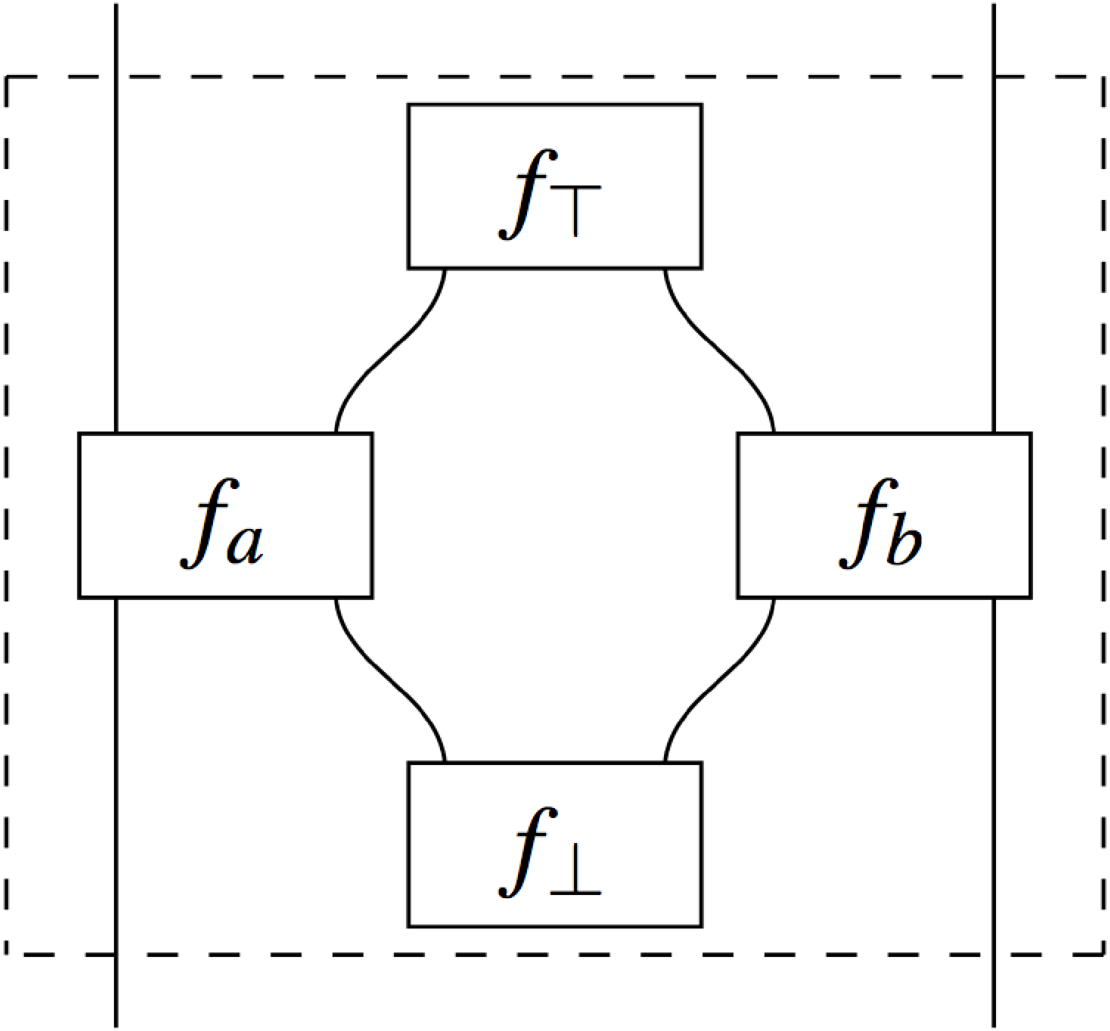
Modification of ‘causality’ in: G. Chiribella, G. M. D’Ariano and P. Perinotti (2009) *Probabilistic theories with purification*. PRA. arXiv:0908.1583

# A-B non-signalling :=

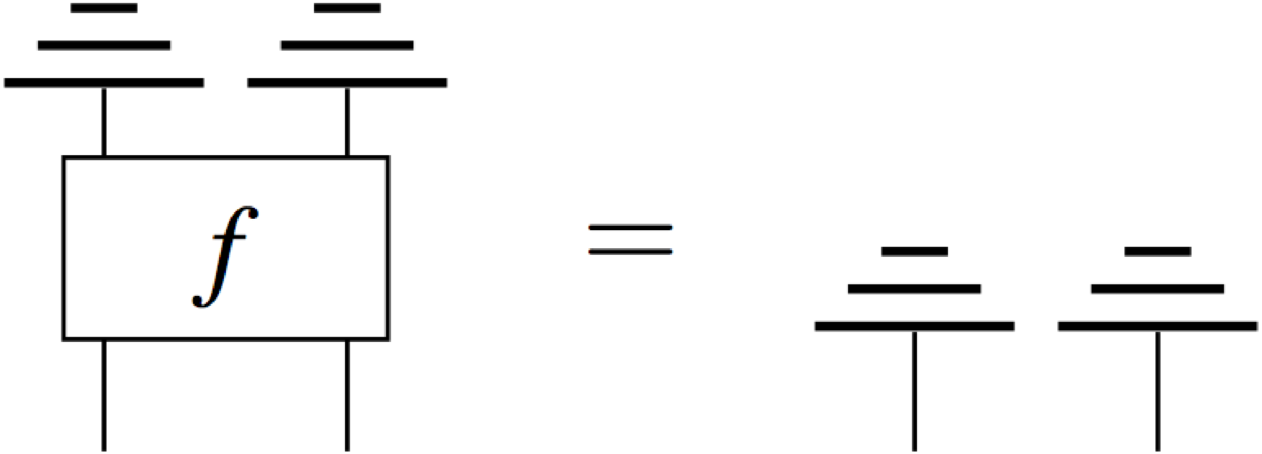


B. Coecke (2014) *Terminality implies no-signalling*. QPL. arXiv:1405.3681

Diamond causal structure :=



# Instances of causality:



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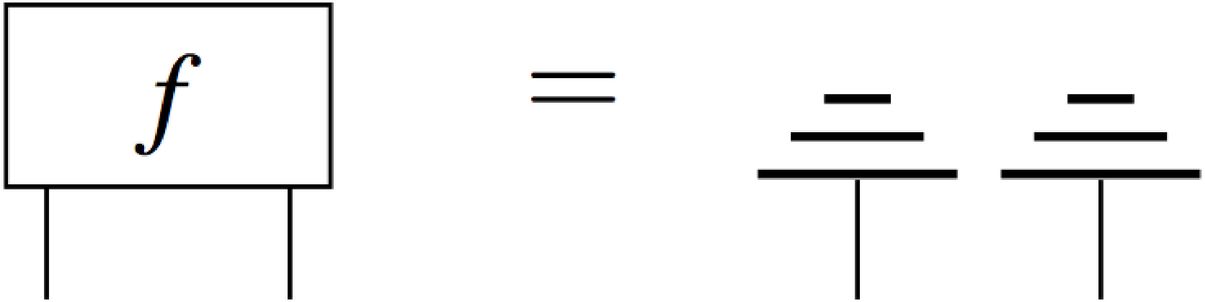
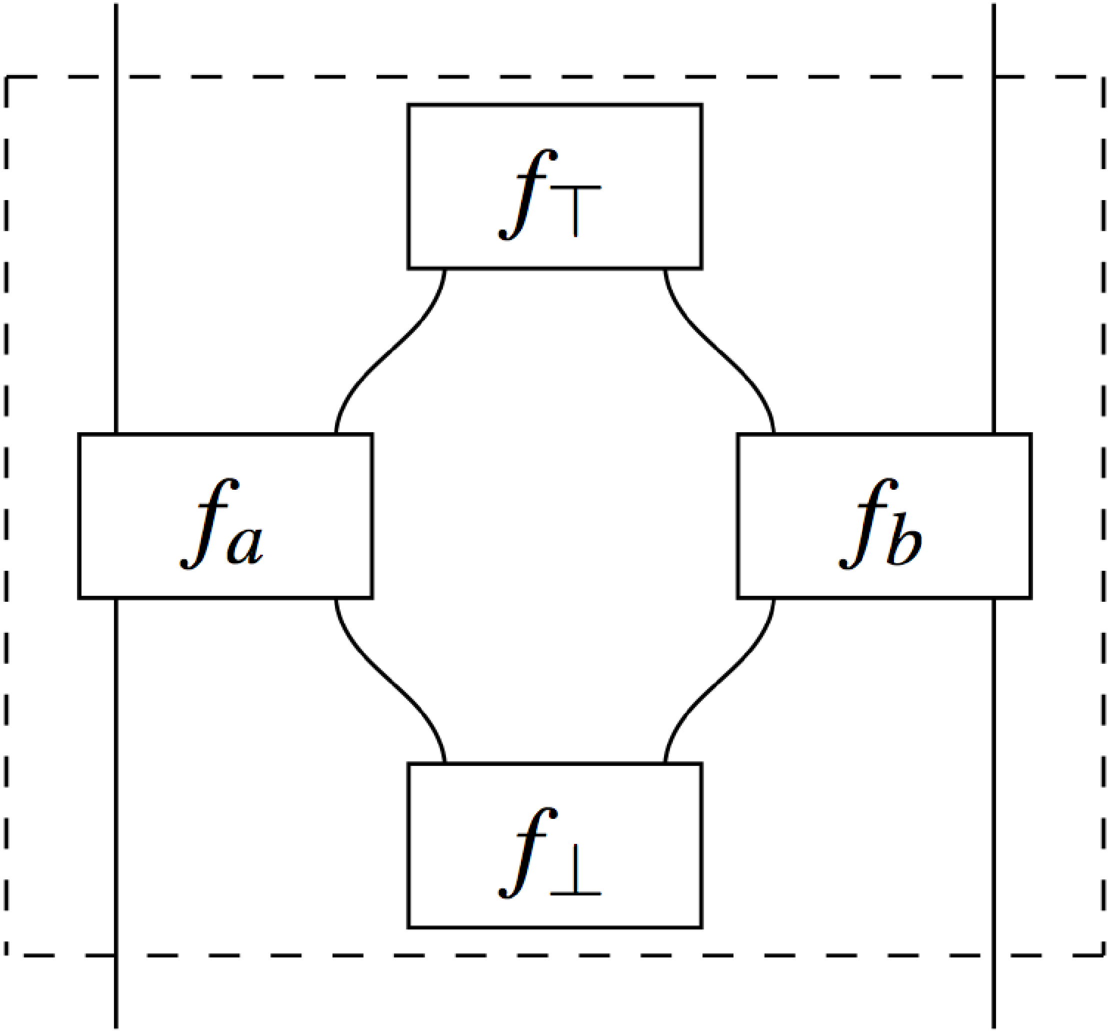


So:



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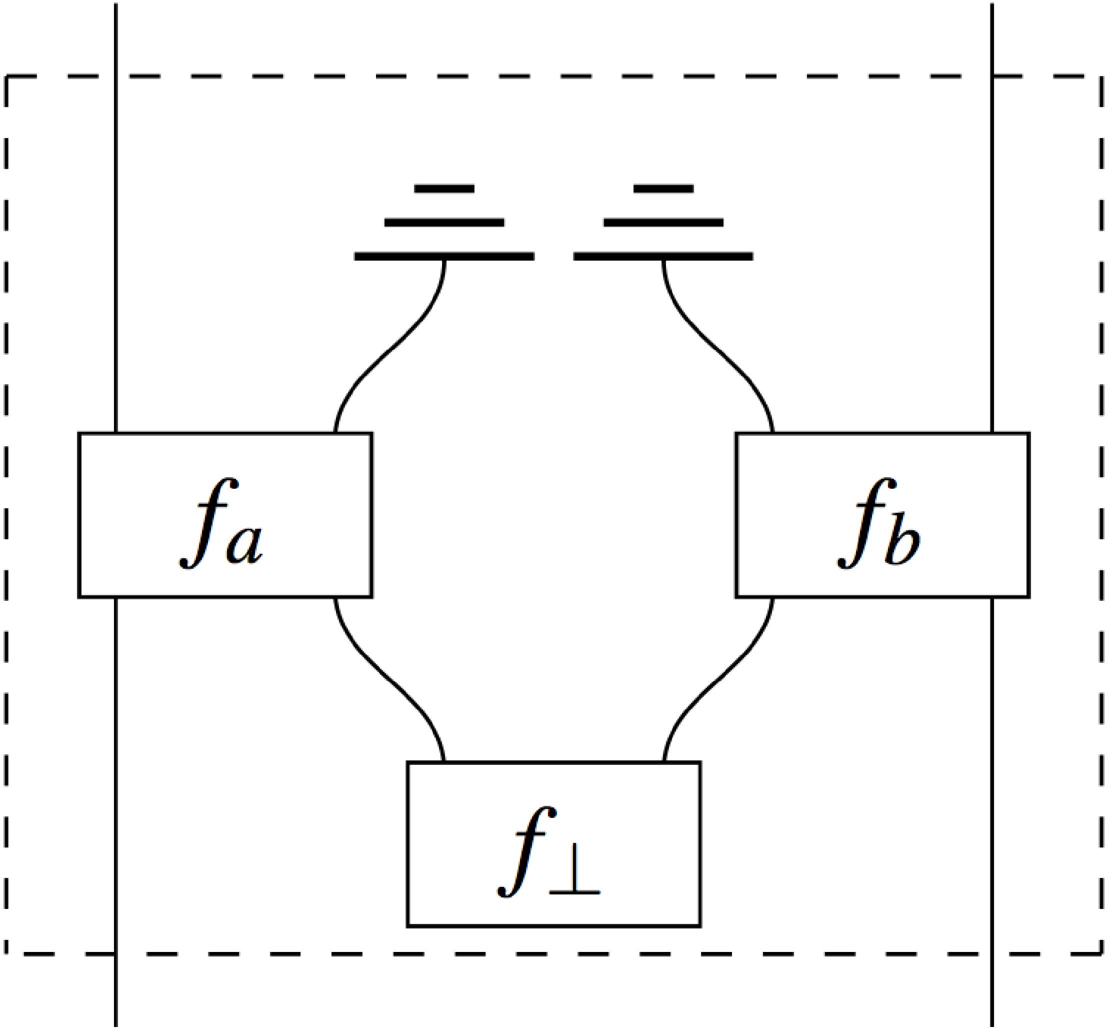

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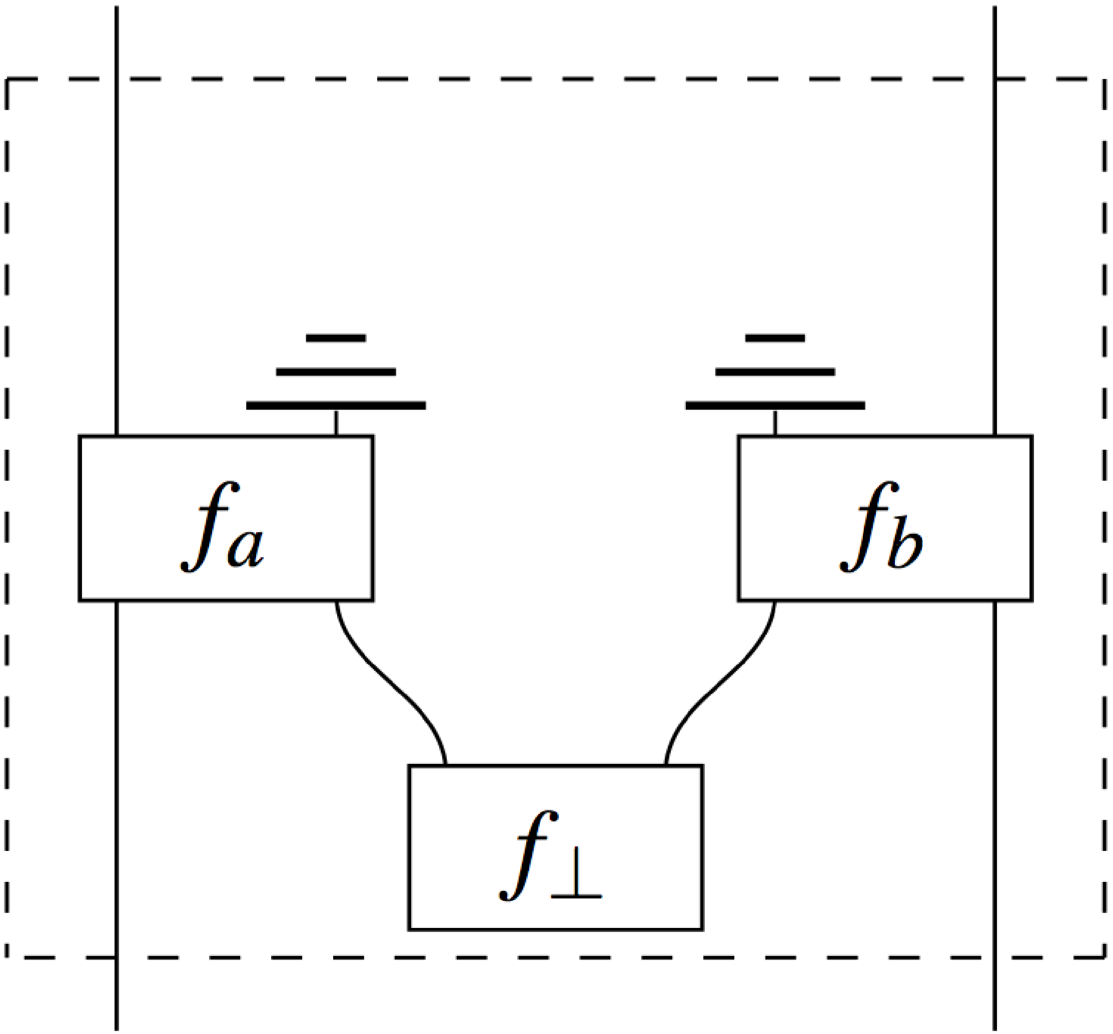
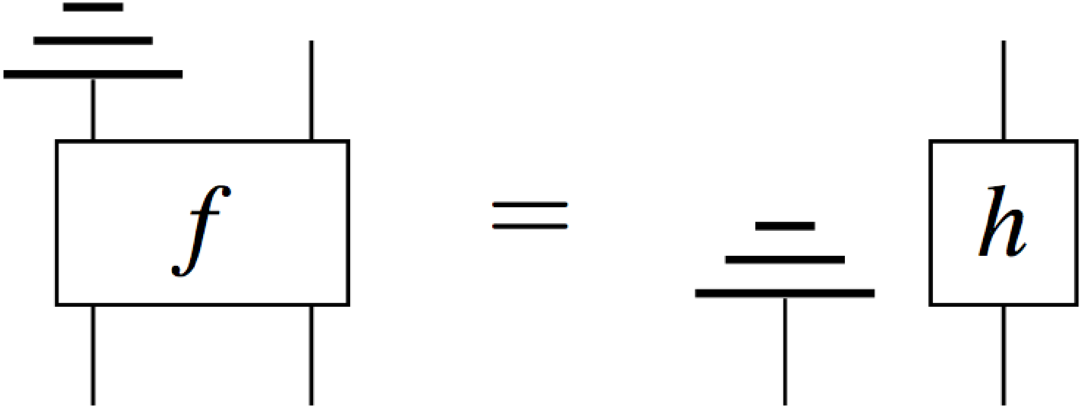
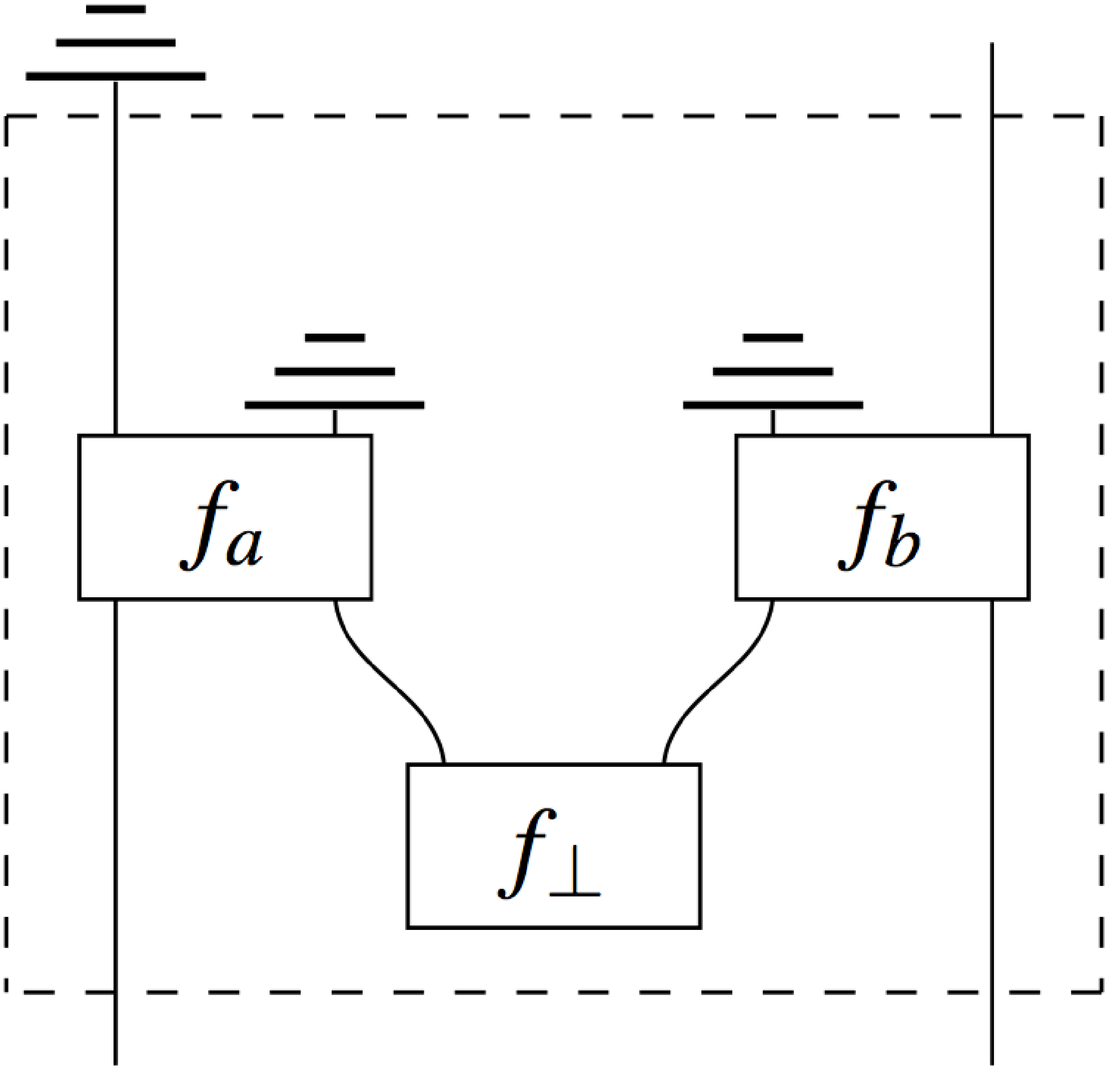


# Recall A-B non-signalling:

$\exists h :$



We have:



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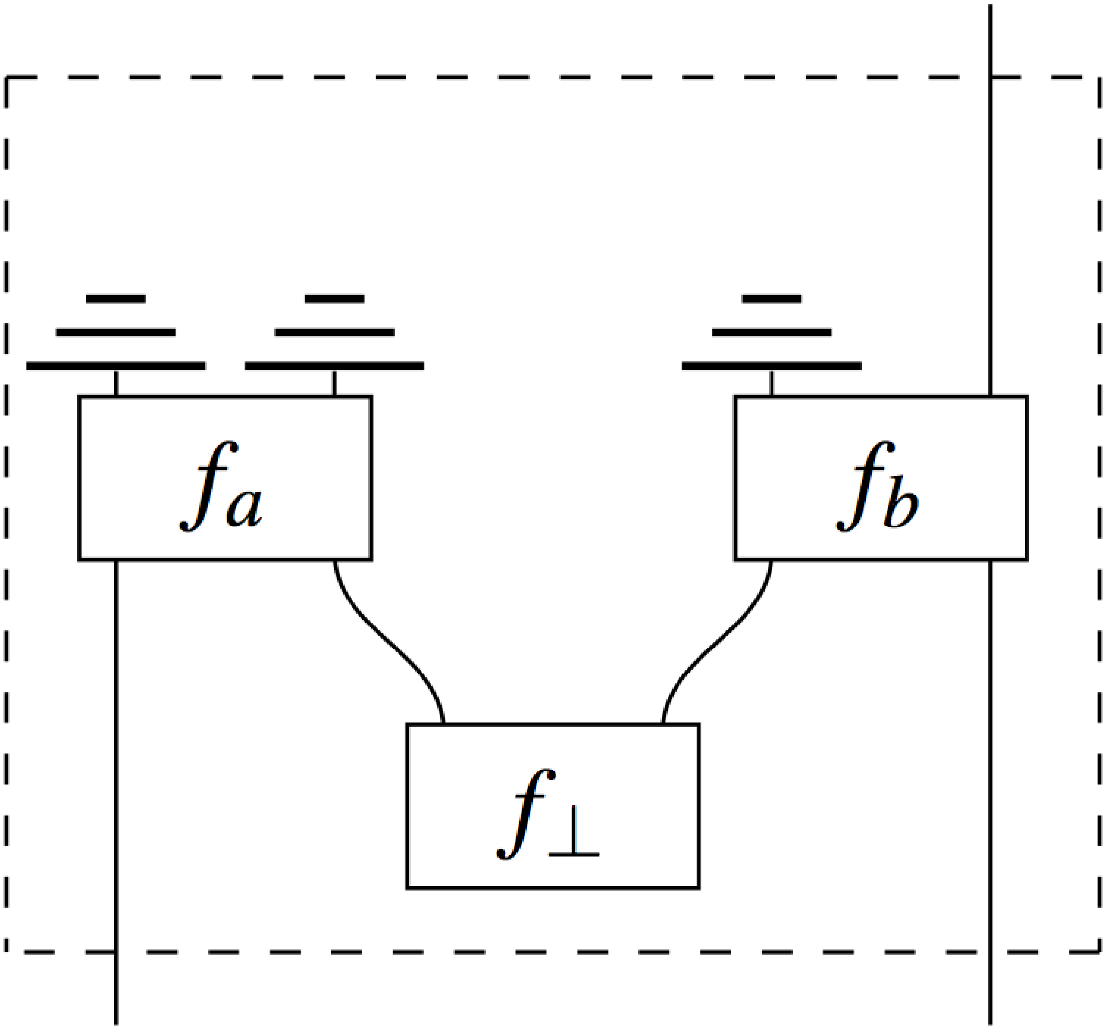

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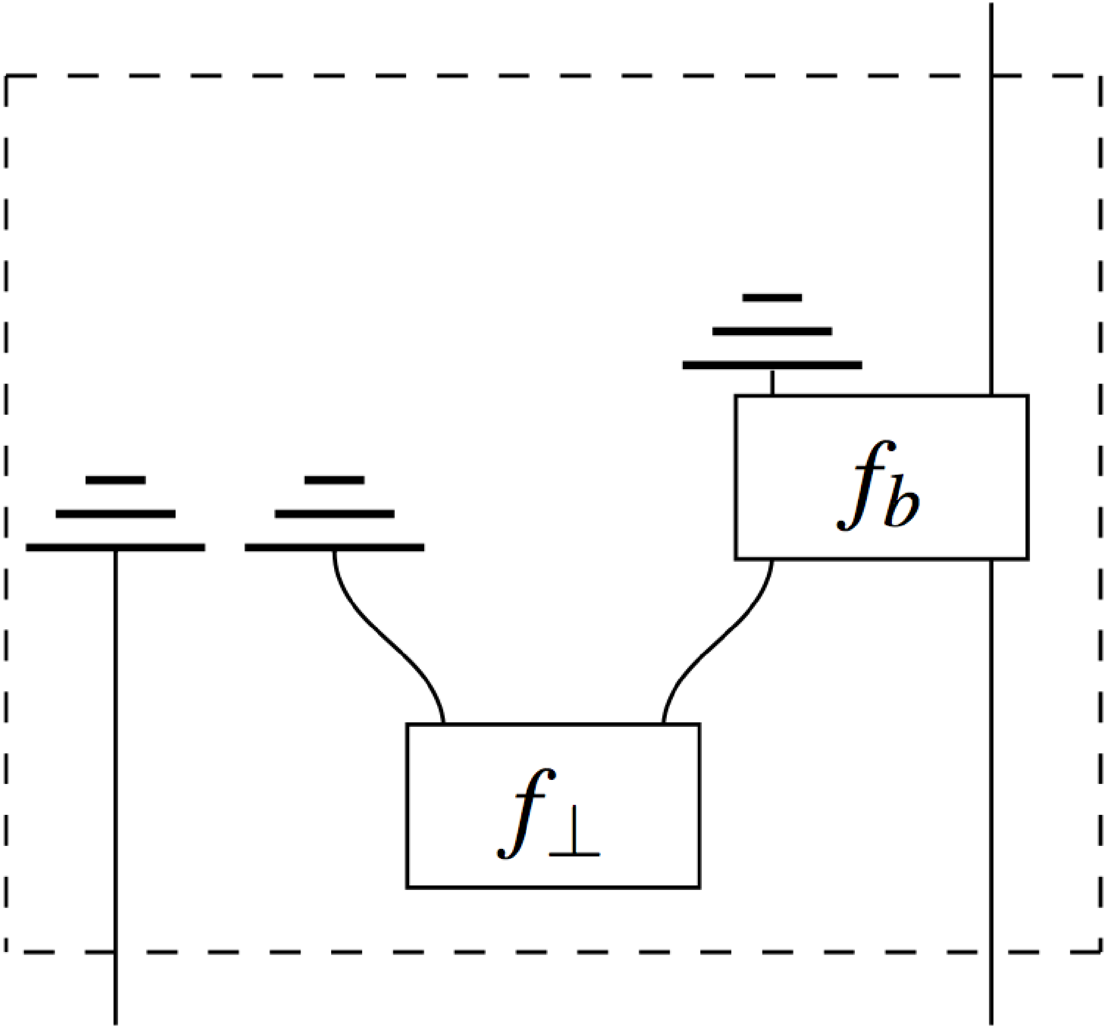

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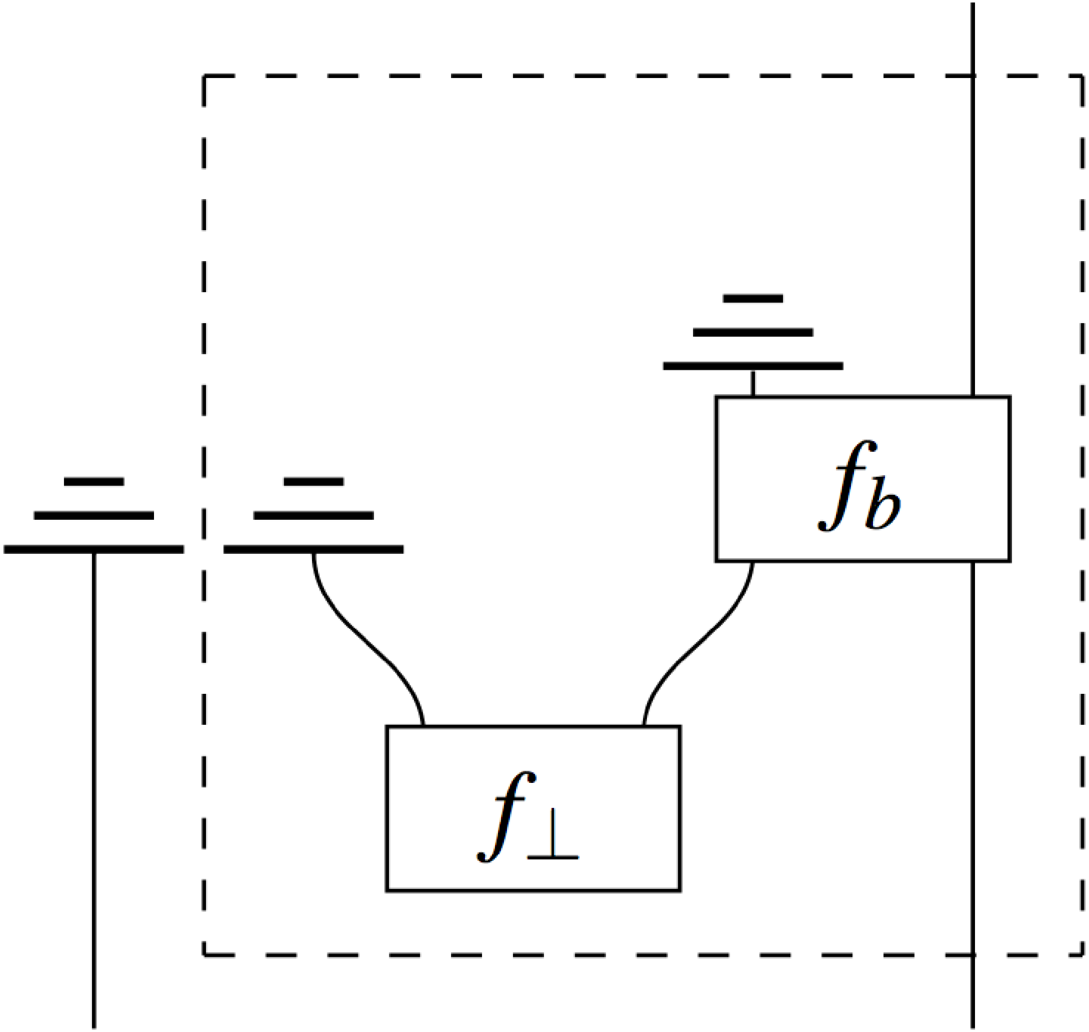
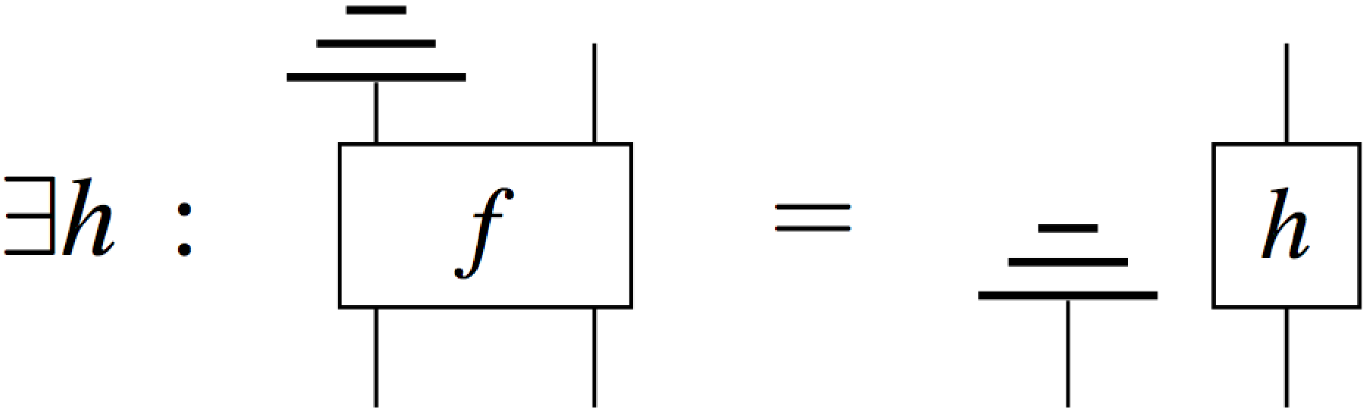
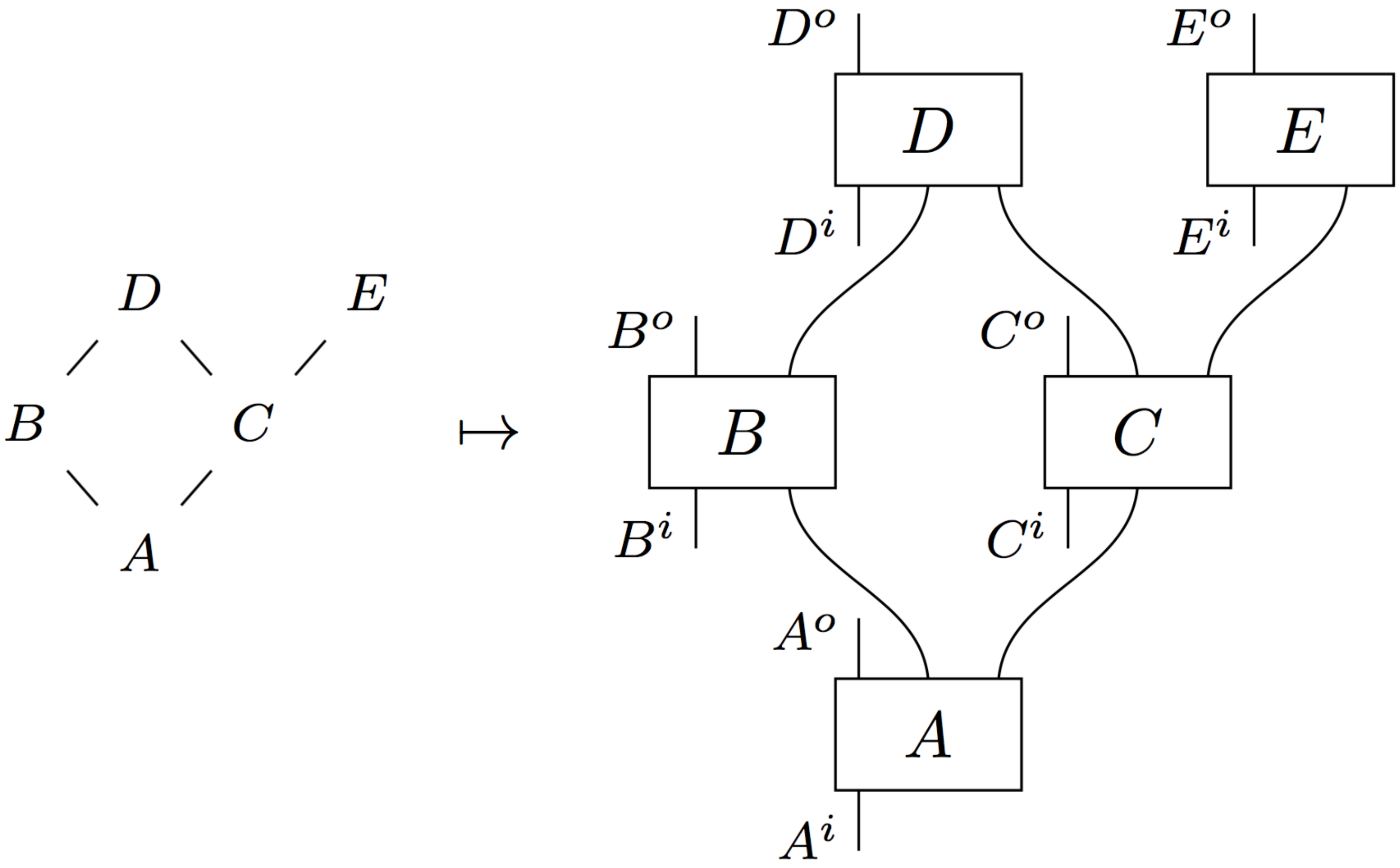


Conversely, take 2nd system trivial in:



# causality ≡ what causal structure is supposed to do



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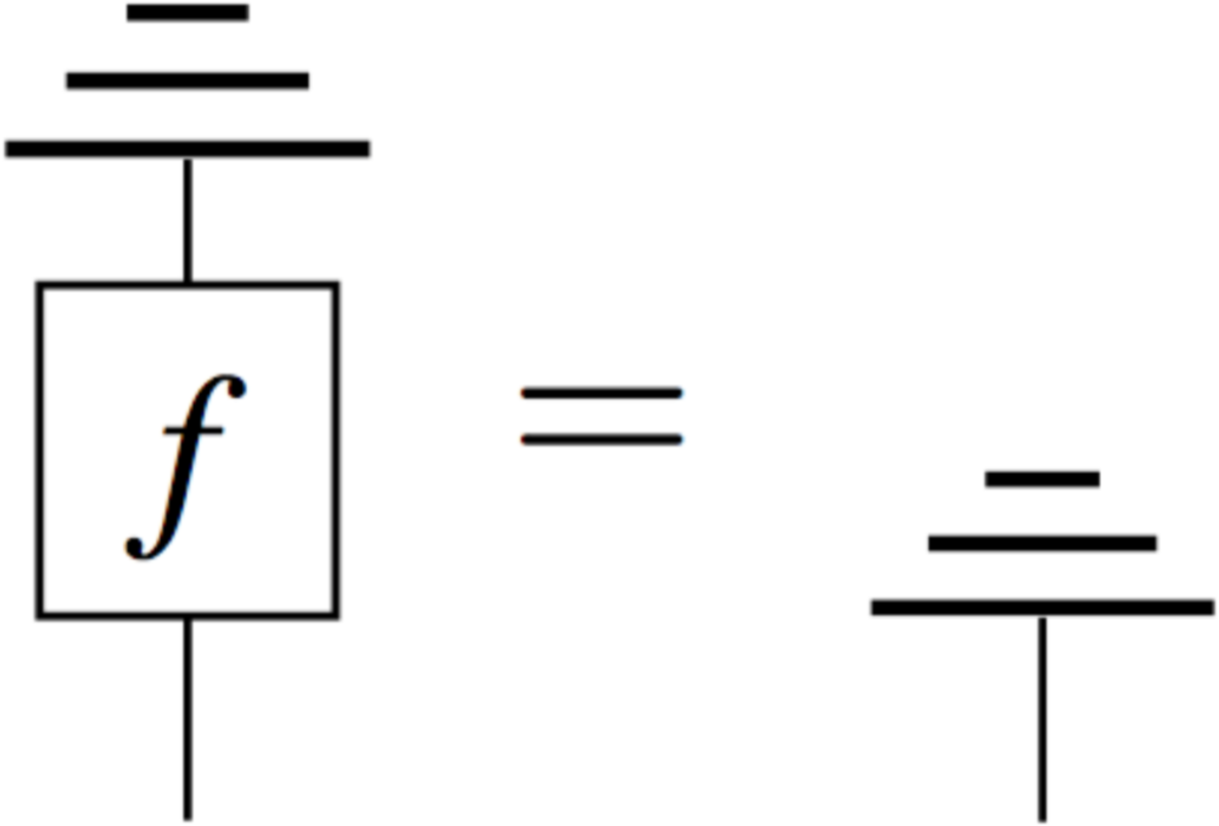
A. Kissinger, M. Hoban & B. Coecke (2017) *Equivalence of relativistic causal structure and process terminality*. arXiv:1708.04118

# time reverse of causal theory = eternal noise



B. Coecke, S. Gogioso & J. Selby (2017) *The time-reverse of any causal theory is eternal noise*. arXiv:1711.05511

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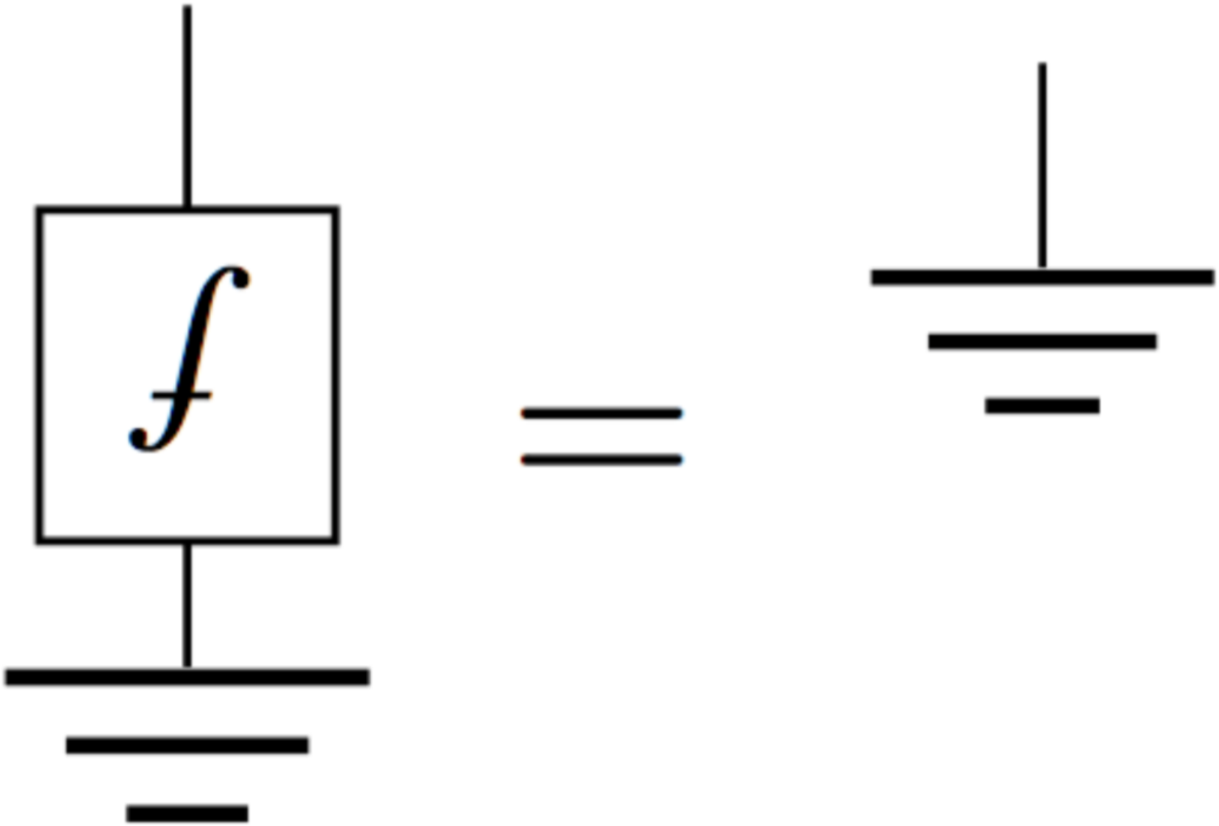

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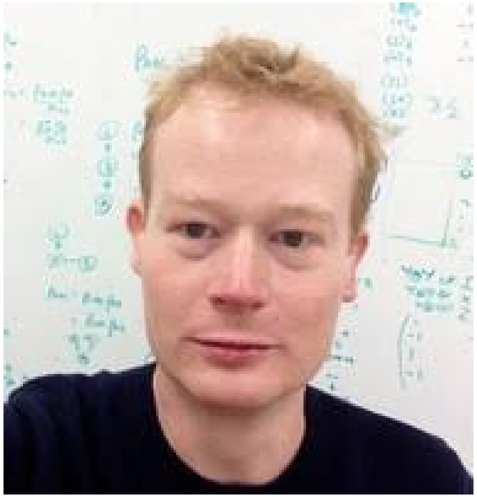
# Oxford wires and boxes

# *– quantum causal faculty –*



Jon Barrett

Aleks Kissinger

Stefano Gogioso

*– IEEE-LiCS 2004 –*

# A categorical semantics for quantum protocols

Samson Abramsky & Bob Coecke

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- Quantum-information-flow structure,

*– IEEE-LiCS 2004 –*

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- ...for general process theories,

– IEEE-LiCS 2018 –

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*– IEEE-LiCS 2018 –*

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– IEEE-LiCS 2018 –

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- ...with automated theorem proving.

*– IEEE-LiCS 2019 –*

# A categorical semantics for time travel

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- Compositional CTCs,

*– IEEE-LiCS 2019 –*

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*– IEEE-LiCS 2019 –*

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- Compositional CTCs,
- ...for general process theories,
- ...with an axiomatic account on paradoxes.

# ...some books are better than others